Qualitative Action Theory

A Comparison of the Semantics of Alternating-Time Temporal Logic and the Kutschera-Belnap Approach to Agency

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Abstract. Qualitative action theory deals with purely qualitative descriptions and formal representations of agency, i.e., agents and their possibilities for intervening in the causal flow of events. This means that, contrary to game theory, qualitative action theory abstains from any metric evaluation of the outcomes of actions.
In this paper we present and compare two qualitative approaches to action theory that have been discussed in the literature. The first one coming from philosophical action theory is the Kutschera-Belnap approach, which is the semantic basis of so-called Stit-logics. The second approach is the semantics of Alur, Henzinger, and Kupferman's Alternating-time Temporal Logic (ATL). In computer science, ATL has been introduced as an extension of Computational Tree Logic (CTL) to allow for modeling systems that interact with their environment. Surprisingly, although both approaches are very close in spirit, a systematic analysis of the mutual dependencies between these approaches does not exist.
The paper aims at bringing together these two research streams, which seem to have been developed independently in philosophy and computer science. In particular, we will investigate the assumptions with which both approaches may be considered equivalent. Finally, further research on this topic promises interesting results that translate between the approaches presented here.

1 Introduction

Qualitative action theory deals with purely qualitative descriptions and formal representations of agency, i.e., agents and their possibilities for intervening in the causal flow of events. Qualitative theories of agency are typically situated in a setting that is well-known in game-theory: there are agents (or players) and each agent has choices concerning how to act (possible moves in the play), where the set of choices an agent has may depend on the current state. Contrary to game theory, however, qualitative action theory abstains from any metric evaluation of the outcomes of the actions. This means, in particular, that qualitative action theory does not aim at a theory of how to act *rationally* in a specific situation,

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but rather restricts consideration to purely descriptive questions such as: what possibilities does an agent have, and how could the world look like, when all the agents behave in a certain manner.

Qualitative action theory (in the narrow sense) restricts consideration to concepts that are definable by causal notions and by terms describing the possible choices of agents. Examples of such *core concepts* include the concept of *action*, the concept of *bringing-it-about-that*, and the concept of *strategy*. In the wider sense qualitative action theory also takes into account concepts referring to doxastic and/or voluntative aspects of agency, such as *beliefs*, *intentions*, *reasons*, *goals*, and *aims*.

In this paper we focus on two qualitative (core) approaches to action theory discussed in the literature. The first one, coming from philosophical action theory, is the Kutschera-Belnap approach to agency. Historically, the first approach using game-theoretical notions for analyses in philosophical action theory was presented by Lennart Åqvist [3]. Based on Åqvist's and Georg H. von Wright's work on agency and Roman Ingarden's work on causality, Franz von Kutschera developed in the 1980s an approach to agency that took the idea seriously that agency is only explainable in the context of an indeterministic theory (cf. [14]). Kutschera presented formal models for representing agency, and he also developed a semantics for action logics. Further developments were contributed by, among others, Nuel D. Belnap, Brian F. Chellas, John F. Horty, and Michael Perloff. In particular, Belnap contributed many philosophical investigations regarding an indeterminist view of agency. He developed a formal semantics that allows for modeling assertions (as speech acts), promisings, and moral obligations.[1] Based on Kutschera and Belnap's semantics, Ming Xu discussed axiomatizations of action-theoretical concepts. In the literature these logical systems are usually referred to as *stit-logics*.[2]

The second approach discussed in this paper is the semantics of Alur, Henzinger, and Kupferman's Alternating-time Temporal Logic (ATL) [1]. In computer science ATL has been introduced as an extension of Computational Tree Logics (CTL) to allow for modeling systems that interact with their environment. In this context it is worth mentioning that there is a close connection between ATL and M. Pauly's *Coalition Game Logic (CGL)* [15,16], as has been pointed out by Valentin Goranko [11]. In particular, ATL, CGL, and game theory share the assumption of a discrete flow of time, while the Kutschera-Belnap approach (in the sequel abbreviated by *KB-approach*) also allows for dense or continuous flows.

Surprisingly, although the Kutschera-Belnap approach and the ATL approach are very close in spirit, a systematic analysis of the mutual dependencies between these approaches does not exist. This paper aims at bringing together these two research streams, which seem to have developed independently in phi-

[1] For discussions of the Kutschera-Belnap approach and for explications of action-theoretical notions in this approach see [4,5], [10], [6,7,8], [12], and [13,14]. A comprising presentation of the current state of discussion may be found in [9].

[2] Cf. [19,20,21,22,23,24] as well as [17].

losophy and computer science. In particular, we will investigate the assumptions with which both approaches may be considered equivalent theories. Finally, further research on this topic promises interesting results that translate between the semantic approaches presented here and the logics defined in terms of these semantics respectively.

2 The Kutschera-Belnap Approach

The basic idea of the Kutschera-Belnap approach can be briefly sketched in the slogan: 'No agency without real choices'. This means that in order to ascribe agency to agents, we must ascribe to them genuine choices for how to act, i. e., choices by which agents can influence the causal flow of events. These choices are *genuine* in that each agent must be able to refrain from what s/he is actually doing. In particular, each agent can realize one of her/his choices independently of what the other agents do at the same moment. Thus, the Kutschera-Belnap approach implicitly assumes that the causal flow of events is not causally determined: If any event were causally determined (by previous events and/or previous circumstances), we would never be able to ascribe genuine choices to agents.

This indeterministic point of view, then, is modeled by tree-like formal structures, i. e., by structures consisting of a set of nodes (called *moments*) and a binary relation defined on this set, which represents the relation of *being-causally-earlier-than.* This relation allows for branching with respect to the future, but not with respect to the past. A (full) branch of such a 'tree' is called *possible history* and represents one of the many possible courses the world might take. The idea is that the future is causally open (in the sense that it is not causally determined by the present and the past), while the past is causally closed (events that occurred in the past are settled, they cannot be made undone). By acting, persons can influence the future, but not the present or the past. But whether an agent can do something or not depends on current circumstances, and these are subject to changes in time. Thus, it may occur that an agent can do something now, but that s/he can not at some later moment. To represent these intuitions within the basic tree-like models, one assigns to each agent at each moment a set of (possible) choices such that each choice is consistent with the choices of all the other agents.

These ideas are captured by the following formal definitions.

Definition 2.1 (Tree). A *tree* is an ordered pair $\mathcal{B} = \langle \text{Mom}, \prec \rangle$ consisting of a non-void set Mom (the set of *moments*) and an irreflexive, transitive, and linear-to-the-left relation $\prec$ on Mom (the relation of *earlier-than*). A maximal $\prec$-chain is said to be a *history* in $\mathcal{B}$, and the set of all histories of $\mathcal{B}$ is denoted by His. For each moment $m \in \text{Mom}$, let

$$\text{His}\,\langle m \rangle := \{\, h \in \text{His} \,:\, m \in h \,\}$$

denote the set of histories that pass through moment m. Histories h and h' are said to be *undivided* at moment m, $h \perp_m h'$, if there exists a moment $m' \in h \cap h'$

with $m \prec m'$. If h and h' pass through m and if there does not exist any $m' \in h \cap h'$ with $m \prec m'$, then h and h' are said to *split* at m.

Definition 2.2 (Agent Tree). An *agent tree* is a triple $\mathcal{C} = \langle \mathcal{B}, \mathrm{Ag}, \mathrm{Ch} \rangle$, where $\mathcal{B}$ is a tree, Ag is a non-void set of *agents*, and Ch is a map that assigns to each agent $\alpha \in \mathrm{Ag}$ and each moment m of $\mathcal{B}$ a partition $\mathrm{Ch}_\alpha\langle m \rangle$ of $\mathrm{His}\,\langle m \rangle$ such that the following conditions are satisfied:

(a) If $h \in X \in \mathrm{Ch}_\alpha\langle m \rangle$ and if h and h' are undivided at m, then h' too is in X.

(b) Let m be a moment of $\mathcal{B}$ and suppose that χ is a map that assigns to each agent α an element $\chi(\alpha) \in \mathrm{Ch}_\alpha\langle m \rangle$. Then there exists a history h that is contained in each $\chi(\alpha)$, i. e.,

$$\bigcap_{\alpha \in \mathrm{Ag}} \chi(\alpha) \neq \emptyset.$$

The elements of $\mathrm{Ch}_\alpha\langle m \rangle$ are said to be the *(momentary) choices* of agent α at moment m, and $\mathrm{Ch}_\alpha\langle m \rangle$ is said to be the *choice set* of α at m. An agent α has *non-vacuous choice* at moment m if $\mathrm{Ch}_\alpha\langle m \rangle \neq \{\mathrm{His}\,\langle m \rangle\}$, i. e., if α has at least two choices at m.

By saying that each agent's choice set forms a partition, we postulate that at each moment each agent chooses exactly one of her/his alternatives. Condition (a) of definition 2.2 means that an agent cannot separate histories that are undivided. Finally, by condition (b), each agent can choose an alternative in her/his choice set independently of the alternatives chosen by all the other agents (at the same moment). In particular, at a given moment m, no agent can prevent another agent from choosing any of her/his alternatives (at that moment).

Given a moment m and a history $h \in \mathrm{His}\,\langle m \rangle$, let $\mathrm{ch}_\alpha(m, h)$ denote the unique element of $\mathrm{Ch}_\alpha\langle m \rangle$ that contains h. This means that $\mathrm{ch}_\alpha(m, h)$ is the choice agent α takes in history h at moment m. An agent tree is said to be *agent-complete* if, for all moments $m \in \mathrm{Mom}$ and each pair of histories $h, h' \in \mathrm{His}\,\langle m \rangle$, it holds:

$$h' \in \bigcap_{\alpha \in \mathrm{Ag}} \mathrm{ch}_\alpha(m, h) \implies h \perp_m h'.$$

The condition of agent completeness was first discussed by Franz von Kutschera [14]. It may be read as: 'No splitting of the tree without the involvement of at least one of the agents.'

3 Alternating-Time Temporal Logic

Alternating-time temporal logic has been introduced to enrich the expressive power of computation tree logics (CTL) for model checking purposes. While CTL is considered a suitable representation for *closed* reactive systems, that is, systems that are completely determined by their current state, ATL aims at *open* systems, that is, systems that allow for interaction with their environment.

Thereto, Alur, Henzinger, and Kupferman [1] introduce the concept of *alternating transition system*, which extends the concept of *transition system* as discussed in CTL. The difference between CTL transition systems and alternating transition systems is characterized as follows: 'While in ordinary transitions systems, each transition corresponds to a possible step of the system, in alternating transition systems, each transition corresponds to a possible move in the game between the system and the environment' [2].

Definition 3.1 (Alternating Transition Frame). An *alternating transition frame* (abbr. by *ATF*) is a triple $\mathcal{F} = \langle \Sigma, Q, \delta \rangle$, where

(a) Σ is a (non-void) set of *agents*,

(b) Q is a (non-void) set of *states*, and

(c) $\delta \colon Q \times \Sigma \longrightarrow 2^{2^Q}$ is a map that assigns to each state q and each agent α a non-void set of choices, $\delta(q, \alpha)$, i. e., each choice is a set of possible next states,

such that for each state q and for each family $(Q_\alpha)_{\alpha \in \Sigma}$ of choices at q, i. e., $Q_\alpha \in \delta(q, \alpha)$, there exists exactly one state q^* with

$$q^* \in \bigcap_{\alpha \in \Sigma} Q_\alpha .$$

In the sequel, the function δ will be referred to as the *transition function.*

Some notations: Let q and q' be states of an ATF $\mathcal{F}$ and let α be an agent. State q' is said to be an *α-successor* of q if there exists a $Q' \in \delta(q, \alpha)$ with $q' \in Q'$. The set of all α-successors is denoted by $\mathrm{Succ}(q, \alpha)$. State q' is a *successor* of q if, in state q, each agent α has a choice Q_α containing q'. The heuristics of this definition is that q' is a successor of q if and only if, in state q, all the agents of $\mathcal{F}$ can cooperate in such a way that q' becomes the next state.

A *(full) computation* of $\mathcal{F}$ is an infinite sequence of states, $\lambda = (q_i)_{i \in \mathbb{N}}$, where each q_{i+1} is a successor of q_i. A *finite computation* (of length n) is an initial segment $\gamma = (q_1, \ldots, q_n)$ of a full computation. For a finite computation γ let n_γ be the length of γ. A *q-computation* is a computation starting in state q. In the sequel, $\lambda[i]$ will denote the i-th state of λ. The set of all (full) computations of $\mathcal{F}$ will be denoted by $\Lambda_{\mathcal{F}}$ and the set of all finite computations by $\Gamma_{\mathcal{F}}$.

The following example (cf. [1]) may help to illustrate the notions just introduced.

Example 3.2. Consider a system S with two processes α and β. In each state of the system, process α determines the truth value of proposition x and likewise process β that of y. We will assume that the system is completely described by propositions x and y, i. e., $Q = \{q, q_x, q_y, q_{xy}\}$, where q_x denotes the state in which x is true in the system, but y is not, etc. The transition function of the system is defined as follows: If S is in a state, in which x is false, α is free to leave the truth value of x unchanged or to change it to true. Otherwise, α leaves x unchanged. Similarly, if y is false, β can leave the value of y unchanged or make it true, and if y is true, β leaves the truth value of y unchanged. This transition function can be defined formally by:

$$\begin{aligned} \delta(q,\alpha) &= \{\{q,q_y\},\{q_x,q_{xy}\}\} & \delta(q,\beta) &= \{\{q,q_x\},\{q_y,q_{xy}\}\} \\ \delta(q_x,\alpha) &= \{\{q_x,q_{xy}\}\} & \delta(q_x,\beta) &= \{\{q,q_x\},\{q_y,q_{xy}\}\} \\ \delta(q_y,\alpha) &= \{\{q,q_y\},\{q_x,q_{xy}\}\} & \delta(q_y,\beta) &= \{\{q_y,q_{xy}\}\} \\ \delta(q_{xy},\alpha) &= \{\{q_x,q_{xy}\}\} & \delta(q_{xy},\beta) &= \{\{q_y,q_{xy}\}\} \end{aligned}$$

4 From Alternating Transition Frames to Agent Trees

In what follows we now investigate the conditions with which the semantics of ATL can be embedded into the Kutschera-Belnap approach. To start with, let $\mathcal{F} = \langle \Sigma, Q, \delta \rangle$ be an ATF that satisfies the following two conditions:

(d) For each agent α and each state q, $\delta(q,\alpha)$ is a partition of $\text{Succ}(q,\alpha)$.

(e) For each agent α and each state q, if q' is an α-successor of q, then q' is a β-successor of q for each agent β.

What is the meaning of these conditions? First, condition (d) seems quite plausible when looking at concrete examples of alternating transition frames. For condition (e), let us assume that q' is an α-successor of q, but that there is an agent β such that q' is not a β-successor of q. From this it follows that there does not exist any computation λ with $\lambda[i] = q$ and $\lambda[i+1] = q'$ for some $i \in \mathbb{N}$. But this means that we can withdraw q' from every $Q \in \delta(q,\alpha)$ without loosing any information about the possible runs of the system. If we do that for every agent at the same time, we obtain an ATF that satisfies condition (e).

For example, we could redefine the transition function of example 3.2 as follows:

$$\begin{aligned} \delta(q,\alpha) &= \{\{q,q_y\},\{q_x,q_{xy}\}\} & \delta(q,\beta) &= \{\{q,q_x\},\{q_y,q_{xy}\}\} \\ \delta(q_x,\alpha) &= \{\{q_x,q_{xy}\}\} & \delta(q_x,\beta) &= \{\{q_x\},\{q_{xy}\}\} \\ \delta(q_y,\alpha) &= \{\{q_y\},\{q_{xy}\}\} & \delta(q_y,\beta) &= \{\{q_y,q_{xy}\}\} \\ \delta(q_{xy},\alpha) &= \{\{q_{xy}\}\} & \delta(q_{xy},\beta) &= \{\{q_{xy}\}\} \end{aligned}$$

This new transition function does not loose any information carried by the old one, but it does satisfy conditions (d) and (e).

Finally, from conditions (d) and (e) it follows that for each agent α and each state q, the choice set $\delta(q,\alpha)$ is a partition of the set of q-successors. In what follows an ATF satisfying these two conditions will be referred to as a *restricted ATF*.

Let now $\mathcal{F} = \langle \Sigma, Q, \delta \rangle$ be an arbitrary ATF. We define a tree $\mathcal{B}^{\mathcal{F}}$ by 'unwinding' $\mathcal{F}$ as follows:

$$\begin{aligned} \mathrm{Mom}^{\mathcal{F}} &:= \quad \Gamma_{\mathcal{F}} \\ \gamma \prec^{\mathcal{F}} \gamma' &:\Longleftrightarrow \gamma[i] = \gamma'[i], \text{ for each } i = 1, \ldots, n_\gamma, \\ &\qquad\ \text{and } n_\gamma < n_{\gamma'}. \end{aligned}$$

Lemma 4.1. *The ordered pair* $\mathcal{B}^{\mathcal{F}} = \langle \mathrm{Mom}^{\mathcal{F}}, \prec^{\mathcal{F}} \rangle$ *is a tree, which has the following properties:*

(a) *There exists a bijective map between the set of computations of* $\mathcal{F}$, $\Lambda_{\mathcal{F}}$, *and the set of histories of* $\mathcal{B}^{\mathcal{F}}$.

(b) *For each finite computation* $\gamma \in \Gamma$, *there exists a bijective map between the set of computations with initial segment* γ *and the set of histories of* $\mathcal{B}^{\mathcal{F}}$ *that pass through 'moment'* γ.

Proof. First, it is quite obvious that $\prec^{\mathcal{F}}$ is an irreflexive, transitive, and linear-to-the-left relation. Second, if λ is a (full) computation of $\mathcal{F}$, then obviously

$$h_\lambda := \{ (\lambda[1], \ldots, \lambda[n]) \ : \ n \in \mathbb{N} \}$$

is a maximal $\prec^{\mathcal{F}}$-chain of $\mathcal{B}^{\mathcal{F}}$. Vice versa, let now h be a history of $\mathcal{B}^{\mathcal{F}}$. Then h is a maximal $\prec^{\mathcal{F}}$-chain, i. e., a maximal subset of $\Lambda_{\mathcal{F}}$ that is linearly ordered by $\prec^{\mathcal{F}}$. Let $\gamma = (q_1, \ldots, q_n) \in h$ be chosen arbitrarily. Then define $\lambda_h[1] := q_1, \ldots, \lambda_h[n] := q_n$. Since q_n has at least one successor, $(q_1, \ldots, q_n)$ cannot be the maximal element of h. Hence there exists a $\gamma' = (q'_1, \ldots, q'_m) \in h$ with $\gamma \prec^{\mathcal{F}} \gamma'$. Extend λ_h by setting $\lambda_h[n+1] := q'_{n+1}, \ldots, \lambda_h[m] := q'_m$. By this step-wise construction, one finally obtains a full computation λ_h.
It can readily be checked that the assignment $h \mapsto \lambda_h$ is the inverse of the mapping $\lambda \mapsto h_\lambda$. From this both claims (a) and (b) follow immediately. □

Let now $\mathcal{F} = \langle \Sigma, Q, \delta \rangle$ be a restricted ATF. For a finite computation γ, let $\Lambda(\gamma)$ be the set of all full computations λ that have γ as initial segment. Define

$$\begin{aligned} \mathrm{Ag}^{\mathcal{F}} &:= \quad \Sigma \\ X \in \mathrm{Ch}^{\mathcal{F}}_\alpha \langle \gamma \rangle &:\Longleftrightarrow \text{there exists a } Q \in \delta(\gamma[n_\gamma], \alpha) \text{ such that} \\ &\qquad X = \{ h_\lambda \ : \ \lambda \in \Lambda(\gamma) \text{ and } \lambda[n_\gamma + 1] \in Q \}. \end{aligned}$$

Theorem 4.2. *For each restricted ATF* $\mathcal{F} = \langle \Sigma, Q, \delta \rangle$, *the triple*

$$\mathcal{C}^{\mathcal{F}} = \left\langle \mathcal{B}^{\mathcal{F}}, \mathrm{Ag}^{\mathcal{F}}, \mathrm{Ch}^{\mathcal{F}} \right\rangle$$

is an agent-complete agent tree.

Proof. From conditions (d) and (e) it follows that each $\delta(q, \alpha)$ is a partition of the set of successors of q. Let γ be a finite computation of $\mathcal{F}$. Then, by applying lemma 4.1(b), we immediately verify that each $\mathrm{Ch}^{\mathcal{F}}_\alpha \langle \gamma \rangle$ is a partition of the set of histories of $\mathcal{B}^{\mathcal{F}}$ that pass through γ. Conditions (a) and (b) of definition 2.2 are easy to check. Finally, $\mathcal{C}^{\mathcal{F}}$ is agent-complete, since for each family $(Q_\alpha)_{\alpha \in \Sigma}$ with $Q_\alpha \in \delta(q, \alpha)$, there exists at most one state $q^* \in \bigcap_{\alpha \in \Sigma} Q_\alpha$. □

5 From Agent Trees to Alternating Transition Frames

Following we will establish an embedding of the semantics of ATL into the KB-approach to agency. The first step in the definition of this embedding is to represent the ATL concept of state in the framework of the KB-approach. However, there does not exist any straight-forward way of defining the notion of state in terms of moments.

To see this, let us assume that we aim at describing a system S with a state set Q. Each $q \in Q$, then, corresponds to a complete description of the system at some time-point. However, when we look at the tree whose branches are the possible computations of the system (as we did in the previous section) the information about possible states of the system has disappeared. Clearly, at each moment (in the sense defined above), the system is in a certain (total) state, but we are *not* able to identify moments that are in the same state.[3]

Definition 5.1 (State Tree). A *state tree* is an ordered triple $\mathcal{B} = \langle \mathrm{Mom}, \prec, \mathrm{Tot} \rangle$ consisting of a non-void set of moments, Mom, an irreflexive, transitive, and linear-to-the left relation on Mom, $\prec$, and a partition Tot of Mom.

The elements of Tot are referred to as *total states*. We say that moments m and m' are in the same (total) state if there exists a $t \in \mathrm{Tot}$ such that m and m' are contained in t. For each moment m, let t_m denote the unique total state that contains m.[4] It is worth noting that a total state may have different pasts, while a moment can only have exactly one past.

In what is to follow, we will restrict consideration on discrete trees, more precisely, on trees where each history is order-isomorphic to the set of natural numbers. Such trees will be referred to as *trees over* $\mathbb{N}$. In a tree over $\mathbb{N}$ each moment m has an immediate successor in each history h passing through moment m, which will be denoted by m_h^*.

Let $\mathcal{B}$ be a state tree, and let m be a moment of $\mathcal{B}$. We define

$$\mathrm{Tot}^* \langle m \rangle := \{ t_{m_h^*} \ : \ h \in \mathrm{His} \langle m \rangle \},$$

the set of *possible next total states* at moment m.

[3] Here and in what is to follow we adopt the following terminology: States correspond to (maybe incomplete) momentary descriptions of a system, while total states correspond to complete momentary descriptions. Thus, states in the sense of the ATL semantics are total states in the sense of the terminology used here.

[4] The concept of state may be introduced in terms of total states as follows: A *state* is a subset of Mom that can be written as a union of total states, i. e.,

$$s \in \mathrm{Stat} \iff \text{there is a } \tau \subseteq \mathrm{Tot} \text{ with } s = \bigcup \tau.$$

Note that this enables us to speak about the inconsistent state, which is distinct from each total state. Furthermore, Tot and Stat are subsets of 2^{Mom}, and each total state is a state.

Definition 5.2. A state tree is said to be *uniform* if for each $t \in \mathrm{Tot}$ and all $m, m' \in t$,

$$\mathrm{Tot}^* \langle m \rangle = \mathrm{Tot}^* \langle m' \rangle .$$

The underlying idea of uniformity is that the partitioning of moments into total states is respected by the successor relation, i. e., that if m and m' are in the same total state, then m and m' have the same possible next total states. From the point of view of the Kutschera-Belnap approach, uniformity seems a very restrictive condition.

If $\mathcal{F}$ is an ATF, then we can use the defintions of section 4 to define a state tree $\mathcal{B}^{\mathcal{F}} = \left\langle \mathrm{Mom}^{\mathcal{F}}, \prec^{\mathcal{F}}, \mathrm{Tot}^{\mathcal{F}} \right\rangle$ by

$$\begin{aligned} \gamma \simeq \gamma' :\Longleftrightarrow \;& \gamma[n_\gamma] = \gamma'[n_{\gamma'}] \\ \mathrm{Tot}^{\mathcal{F}} := \;& \Gamma_{\mathcal{F}}/{\simeq}. \end{aligned}$$

Note that there exists a bijective map between the state set of $\mathcal{F}$, Q, and the set $\mathrm{Tot}^{\mathcal{F}}$.

Lemma 5.3. *Let $\mathcal{F}$ be an ATF. Then the tree $\mathcal{B}^{\mathcal{F}}$ is uniform.* □

Definition 5.4 (Agent state tree). An *agent state tree* is a triple $\mathcal{C} = \langle \mathcal{B}, \mathrm{Ag}, \mathrm{Ch} \rangle$, where $\mathcal{B}$ is a state tree, Ag is a set of agents, and Ch is a choice map as specified in definition 2.2.

Obviously, $\mathrm{Ch}_\alpha\langle m \rangle$ induces a partition of the set of successor moments of m, $\{\, m_h^* \,:\, m \in h \}$. But, as can be seen from simple examples, $\mathrm{Ch}_\alpha\langle m \rangle$ does not induce a partition of $\mathrm{Tot}^* \langle m \rangle$. Therefore, we need to extend the uniformity condition of the previous paragraph. Define

$$\mathrm{Tot}^*_\alpha \langle m, X \rangle := \{\, t_{m_h^*} \,:\, m \in h \in X \} = \{\, t_{m_h^*} \,:\, h \in \mathrm{His}\, \langle m \rangle \cap X \}$$

where $X \in \mathrm{Ch}_\alpha\langle m \rangle$, i. e., $\mathrm{Tot}^*_\alpha \langle m, X \rangle$ is the set of possible next total states in case that α chooses X at moment m.

Definition 5.5. An agent state tree over $\mathbb{N}$ is said to be *uniform* if the choice map Ch respects uniformity, i. e., if for each agent α, each total state t, and each pair of moments $m, m' \in t$,

$$\{\, \mathrm{Tot}^*_\alpha \langle m, X \rangle \,:\, X \in \mathrm{Ch}_\alpha\langle m \rangle \,\} = \{\, \mathrm{Tot}^*_\alpha \langle m', X' \rangle \,:\, X' \in \mathrm{Ch}_\alpha\langle m' \rangle \,\} .$$

Note that if an agent state tree is uniform, its underlying state tree is so, too. This follows from the fact that $\mathrm{Tot}^* \langle m \rangle = \bigcup_{X \in \mathrm{Ch}_\alpha\langle m \rangle} \mathrm{Tot}^*_\alpha \langle m, X \rangle$.

Lemma 5.6. *Let $\mathcal{F}$ be a restricted ATF. Then the agent state tree $\mathcal{C}^{\mathcal{F}}$ (as defined in this and the previous section) is uniform.* □

Let now $\mathcal{C}$ be a uniform agent state tree over $\mathbb{N}$. We set

$$\begin{aligned}\Sigma^{\mathcal{C}} &:= \mathrm{Ag}\\ Q^{\mathcal{C}} &:= \mathrm{Tot}\\ \delta^{\mathcal{C}}(t,\alpha) &:= \{\, \mathrm{Tot}^*_\alpha \langle m, X\rangle \;:\; X \in \mathrm{Ch}_\alpha\langle m\rangle \,\}\end{aligned}$$

where m is an arbitrarily fixed element of t. This definition is well defined since the tree $\mathcal{C}$ is uniform. Note that $\mathrm{Tot}^*_\alpha\langle m, X\rangle$ is a set of total states. Hence $\mathrm{Tot}^*_\alpha\langle m, X\rangle \in 2^{\mathrm{Tot}}$, and thus $\delta^{\mathcal{C}}(t,\alpha) \in 2^{2^{\mathrm{Tot}}}$.

We are now ready to state our second theorem:

Theorem 5.7. *Let $\mathcal{C}$ be an agent-complete and uniform agent state tree over $\mathbb{N}$. Then*

$$\mathcal{F}^{\mathcal{C}} = \left\langle \Sigma^{\mathcal{C}}, Q^{\mathcal{C}}, \delta^{\mathcal{C}} \right\rangle$$

is a restricted alternating transition frame.

Proof. There is almost nothing left to be proven. Let $t \in Q^{\mathcal{C}}$ be a (total) state, and let $(Q_\alpha)_{\alpha\in\Sigma^{\mathcal{C}}}$ be a family with $Q_\alpha \in \delta^{\mathcal{C}}(t,\alpha)$. Choose an arbitrary $m \in t$. Then, for each Q_α, there exists a $\chi(\alpha) \in \mathrm{Ch}_\alpha\langle m\rangle$ with $Q_\alpha = \mathrm{Tot}^*_\alpha\langle m, \chi(\alpha)\rangle$. By applying condition 2.2, there exists a history h that is contained in each $\chi(\alpha)$. Since the tree is agent-complete, h is uniquely determined up to undivided histories. This means that $m^*_{h'} = m^*_h$, for each history $h' \in \bigcap_\alpha \chi(\alpha)$. Since the $\delta^{\mathcal{C}}(t,\alpha)$ do not depend on the particular choice of m in t, there exists exactly one total state that is contained in each Q_α, namely $t_{m^*_h}$. That the frame $\mathcal{F}^{\mathcal{C}}$ is restricted follows from the fact that each $\mathrm{Ch}_\alpha\langle m\rangle$ is a partition of the set of histories that pass through moment m. □

Finally, if an agent state tree $\mathcal{C}$, as specified in the theorem, is a 'forest' such that each total state is realized in exactly one of its root moments, then there exists a bijection between the set of histories of $\mathcal{C}$ and the set of computations of $\mathcal{F}^{\mathcal{C}}$.

6 Summary and Outlook

In this paper we focused on two approaches to qualitative action theory, the Kutschera-Belnap approach and the semantics of alternating-time temporal logic. Though at first glance both approaches are very close in spirit, they could not be found to be equivalent without modifying the basic semantics respectively. If reasonable conditions on alternating transition frames are enforced, these frames can be shown to induce agent trees. Vice versa, agent trees do induce alternating transition frames if they are enriched with the notion of state and if some uniformity conditions are assumed. However, from the point of view of the Kutschera-Belnap approach, these uniformity constraints seem very special. The best interpretation of them is to read ATL choices (i. e., the elements of the agent-dependent transition function) as *action types* in the following sense: Each agent has a repertoire of 'procedures' that can be performed when the system is in a particular state. Whether this procedure can be performed depends on the current state only and not on one of the many possible pasts the system might have passed through to reach this state. Contrary to this, choices in the Kutschera-Belnap approach are assigned with respect to moments, i. e., with respect to the current state and one particular past of that state.

It is also worthwhile to note that in ATL the notion of strategy is defined in a more KB-like manner, i. e., strategies are not defined with respect to single states only, but are defined with respect to finite computations. More precisely, in the KB-approach a *(strict) strategy* of an agent α is a partial function σ that has as its domain a non-void convex subset of Mom, dom(σ), and that assigns to each $m \in \mathrm{dom}(\sigma)$, a choice $\sigma(m) \in \mathrm{Ch}_\alpha\langle m\rangle$. In the ATL-approach a *strategy* of α is a map that assigns to each finite computation γ a choice $Q \in \delta(\gamma[n_\gamma], \alpha)$. The close relationship between these two notions of strategy should now be obvious.

The results presented in this paper provide the start point of an interesting research topic: What are the connections between the *logics* that are defined with respect to the semantic concepts presented here? But an answer to this question would go far beyond the scope of this paper.

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