



**Tutorials for “Formal methods for Java”**  
**Exercise sheet 7**

**Exercise 1: Substitution Lemma**

Let  $M$  be a structure and  $\phi(t)$  be a term or a formula. Prove that

$$M[\phi(t)] = M[x_0 := M[t]][\phi(x_0)]$$

where  $x_0$  is a fresh constant symbol not occurring in  $\phi(t)$ .

**Exercise 2: Soundness of sequent calculus**

Prove that the sequent calculus rule **exists-right** is sound. **Hint:** Use Exercise 1.

**Exercise 3: Skolem formulae**

Let  $\Gamma \Longrightarrow \Delta$  be an unprovable sequent and  $\phi(X)$  a formula. Show that the sequent

$$\Gamma \Longrightarrow \Delta, \exists Y((\exists X \phi(X)) \rightarrow \phi(Y))$$

is provable in the sequent calculus.

**Exercise 4: Completeness of sequent calculus**

Finish the completeness prove of sequent calculus presented in the lecture, i.e. show that for the constructed structure  $M$  and any formula  $\phi$  we have  $M[\phi] = \mathbf{true}$  if and only if  $\phi \in \Gamma$ . **Hint:** Use induction on the number of logical operators in  $\phi$ .