



**Tutorials for “Formal methods for Java”**  
**Exercise sheet 6**

**Exercise 1: Proofs in sequent calculus**

Give proofs for the following sequents in sequent calculus.

(a)  $\Longrightarrow (\exists x \neg P(x)) \vee (\forall x P(x))$ .

(b)  $\Longrightarrow (\exists y \forall x R(x, y)) \rightarrow (\forall x \exists y R(x, y))$ .

(c)  $\Longrightarrow ((\forall x P(x)) \vee (\forall x Q(x))) \rightarrow (\forall x (P(x) \vee Q(x)))$ .

(d)  $\Longrightarrow \exists x [(P(x) \rightarrow Q(x)) \rightarrow (\forall x (P(x) \rightarrow Q(x)))]$ .

(e)  $\Longrightarrow (\forall x \forall y (g(f(x)) = g(f(y)) \rightarrow x = y)) \rightarrow (\forall x \forall y (f(x) = f(y) \rightarrow x = y))$ .

**Exercise 2: Proofs in the KeY system**

Use the KeY prover to prove the sequents from Exercise 1.

**Exercise 3: Logical operators**

From the logical operators **false**,  $\wedge$ ,  $\rightarrow$  and  $\forall$ , all other logical operators are definable. Find formulas defining

(a) **true**,

(b)  $\neg F$

(c)  $F \vee G$

(d)  $\exists x F$

in terms of **false**,  $\wedge$ ,  $\rightarrow$  and  $\forall$ . Prove the validity of these definitions in sequent calculus.