

# Logics, Categories, and Colimits for Artificial Intelligence

T. Mossakowski  
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University of Freiburg  
Department of Computer Science

## Exercise Sheet 12

**Due: January 30, 2009**

### Exercise 12.1 (Functor categories)

Given categories  $\mathbf{C}$  and  $\mathbf{D}$ , prove that  $[\mathbf{C}, \mathbf{D}]$  (functors from  $\mathbf{C}$  to  $\mathbf{D}$  as objects and natural transformations as morphisms) indeed forms a category.

### Exercise 12.2 (Institution comorphisms I)

Spell out the definition of institution comorphisms in terms of the “institutions as functors” definition  $I : \mathbf{Sign} \rightarrow \mathbf{Room}$ .

### Exercise 12.3 (Institution comorphisms II)

Define a model-expansive institution comorphism  $\text{Propositional} \rightarrow \mathcal{ALC}$ .

### Exercise 12.4 (Satisfaction relations as natural transformations)

Show that the satisfaction relation in an institution is a natural transformation  $\models : \text{Gr} \circ \text{Sen} \rightarrow \overline{\text{Gr}} \circ U^{\text{op}} \circ \text{Mod}^{\text{op}}$ , where  $\text{Gr} : \mathbf{Set} \rightarrow \mathbf{Rel}$  and  $\overline{\text{Gr}} : \mathbf{Set}^{\text{op}} \rightarrow \mathbf{Rel}$  map a function to its graph.

### Exercise 12.5 (Comorphisms in HETS)

Experiment with HETS: Translate theories that you have used along comorphisms.

The exercise sheets may and should be worked on in groups of two (2) students. Please write both names on your solution.