

Logics, Categories, and Colimits for Artificial Intelligence

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Exercise Sheet 10

Due: January 16, 2009

Exercise 10.1 (Coproducts I)

Show that

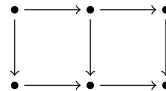
- (a) $[h \circ f, h \circ g] = h \circ [f, g]$
- (b) $(f \amalg g) \circ (k \amalg h) = (f \circ k) \amalg (g \circ h)$

Exercise 10.2 (Coproducts II)

- (a) Let A and B be elements of a partially ordered set, considered as a category. What is the coproduct of A and B ?
- (b) Let P and Q be objects in \mathbf{PoSet} , the category of partially ordered sets. What is the coproduct of P and Q ?

Exercise 10.3 (Pushouts)

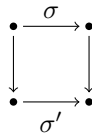
Consider the following diagram:



- (a) Prove that if both squares are pushouts, then the outside rectangle (with top and bottom edges the evident composites) is a pushout.
- (b) Prove that if the outside rectangle and the left-hand square are pushouts and the whole diagram commutes, then the right-hand square is a pushout.
- (c) Try to empirically verify this with some examples in HETS.

Exercise 10.4 (Pushouts of theories and theory morphisms)

Consider a pushout of theories and theory morphisms in a weakly semi-exact institution:



Show that if σ is model-theoretically conservative, then so is σ' .

The exercise sheets may and should be worked on in groups of two (2) students. Please write both names on your solution.