

# Logics, Categories, and Colimits for Artificial Intelligence

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## Exercise Sheet 8

Due: December 19, 2008

### Exercise 8.1 (Theory morphisms)

- (a) Show with HETS that  $\text{Monoid} \rightarrow \text{Nat}$  is a theory morphism.
- (b) Specify that classical extensional parthood has models which use the powerset without the empty set.
- (c) Specify another theory morphism of your choice and prove it with HETS.

### Exercise 8.2 (Functors)

Given a category  $\mathbf{C}$ , what are the functors

(a) from  $\mathbf{1} = \begin{array}{c} \text{id}_A \\ \curvearrowright \\ A \end{array}$  to  $\mathbf{C}$ ?

(b) from  $\mathbf{2} = \begin{array}{c} \text{id}_A \quad \text{id}_B \\ \curvearrowright \quad \curvearrowright \\ A \xrightarrow{f} B \end{array}$  to  $\mathbf{C}$ ?

(c) from  $\mathbf{3} = \begin{array}{c} \text{id}_B \\ \curvearrowright \\ B \\ \swarrow f \quad \searrow g \\ \text{id}_A \quad \curvearrowright \quad \text{id}_C \\ A \xrightarrow{h} C \end{array}$  to  $\mathbf{C}$ ?

### Exercise 8.3 (Institution of theories)

Given an institution  $I$ , obtain  $I^{\text{th}}$  by replacing the category of signatures with the category of theories in  $I$ . Show that this yields an institution.

### Exercise 8.4 (Structured specifications)

- (a) Show that, given a structured specification  $\text{SP}$  and a signature morphism  $\sigma : \Sigma \rightarrow \text{Sig}(\text{SP})$ ,
  - (i)  $\sigma$  is a specification morphism  $\sigma : \text{SP} \text{ hide } \sigma \rightarrow \text{SP}$
  - (ii) the specification morphism is model-theoretically conservative
- (b) Structure some of the specifications that you have written so far and/or that have been used in the lecture using the new structuring mechanisms.

The exercise sheets may and should be worked on in groups of two (2) students. Please write both names on your solution.