

# Qualitative Representation and Reasoning

## Spatial Representation and Reasoning: RCC8 and Topology

Knowledge Representation and Reasoning

January 30, 2006

# RCC8 and Topology – Outline

## RCC8 and Topology

Motivation

RCC8

Topology

Topological Set Constraints

From set constraints to modal logic

## Reasoning with RCC8

# Motivation

We may want to state qualitative relationships between regions in space, for example:

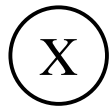
- ▶ “Region  $X$  touches region  $Y$ ”
- ▶ “Germany and Switzerland have a common border”
- ▶ “Freiburg is located in Baden-Württemberg”

# Possible Applications

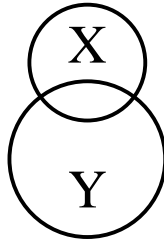
- ▶ This can be useful when only *partial information* is available:
  - ▶ We may know that region  $X$  is *not connected* with region  $Y$  without knowing the shape and location of  $X$  and  $Y$ .
- ▶ We may want to *query* a database:
  - ▶ Show me all countries *bordering* the Mediterranean!
- ▶ We may want to state *integrity constraints*:
  - ▶ An island has to be located *in the interior of* a sea.

# Qualitative Relations Between Regions: RCC8

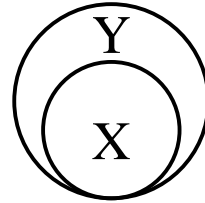
Eight relations between **regions**:



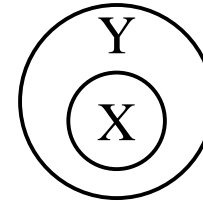
DC(X,Y)



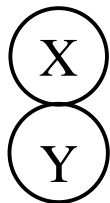
PO(X,Y)



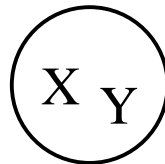
TPP(X,Y)



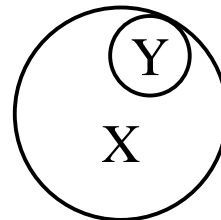
NTPP(X,Y)



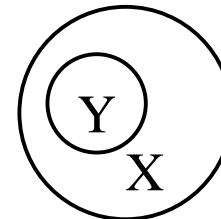
EC(X,Y)



EQ(X,Y)



TPP<sup>u</sup>(X,Y)



NTPP<sup>u</sup>(X,Y)

# Intuition

- ▶ **Regions** are some “*reasonable*” *non-empty subsets of space*.
- ▶ **DC** (disconnected) means that the two regions do not share any point at all.
- ▶ **EC** (externally connected) means that they only share *borders*.
- ▶ **PO** (partially overlapping) means that the two regions share *interior* points.
- ▶ **TPP** (tangential proper part) means that one region is a subset of the other sharing some points on the *borders*.
- ▶ **NTPP** (non-tangential proper part) same, but without sharing any bordering points.

# Questions

- ▶ How can we formalize **regions**?
- ▶ How can we formalize these **relations**?
- ▶ Are they **disjoint** and **exhaustive**?
- ▶ Can we come up with a **composition table**?
- ▶ What is the computational complexity of reasoning with these relations?
- ▶ Can we identify a tractable fragment?

# Point-Set Topology

**Point-set topology** is a mathematical theory that deals with properties of space independent of size and shape.

In topology, we can define notions such as

- ▶ **interior** and **exterior** points of regions,
- ▶ **isolated points** of regions,
- ▶ **boundaries** of regions,
- ▶ **connected components** of regions,
- ▶ **connected regions**,
- ▶ ...

~> Topology seems to be the right formal framework.

# Topology

## Definition

A *topological space* is a pair  $\mathcal{T} = (\mathcal{U}, \mathcal{O})$ , where

- ▶  $\mathcal{U}$  is a nonempty set (the **universe**), and
- ▶  $\mathcal{O}$  is a set of subsets of  $\mathcal{U}$  (the **open sets**)

such that the following conditions hold:

- ▶  $\emptyset \in \mathcal{O}$  and  $\mathcal{U} \in \mathcal{O}$ .
- ▶ If  $O_1 \in \mathcal{O}$  and  $O_2 \in \mathcal{O}$ , then  $O_1 \cap O_2 \in \mathcal{O}$ .
- ▶ If  $(O_i)_{i \in I}$  is a (possibly infinite) family of elements from  $\mathcal{O}$ , then

$$\bigcup_{i \in I} O_i \in \mathcal{O}.$$

**Example:** In Euclidian space, a set  $O$  is open if for each point  $x \in O$  there is a ball surrounding  $x$  that is contained in  $O$ .

# Terminology & Notation

## Definition

- ▶ A set  $N \subseteq \mathcal{U}$  is a *neighborhood* of a point  $x$  if there is an open set  $O \in \mathcal{O}$  such that  $x \in O \subseteq N$ . Let  $X \subseteq \mathcal{U}$  and  $x \in \mathcal{U}$ .
- ▶  $x \in \mathcal{U}$  is an *interior point* of  $X$  if there is a neighborhood  $N$  of  $x$  such that  $N \subseteq X$ .
- ▶  $x \in \mathcal{U}$  is a *touching point* of  $X$  if every neighborhood of  $x$  has a nonempty intersection with  $X$ .
- ▶  $x \in \mathcal{U}$  is a *boundary point* of  $X$  if  $x$  is a touching point of  $X$  and of its complement  $\overline{X}$ .

## Notation:

- ▶  $i(X)$  is the set of *interior points* of  $X$  (the *interior* of  $X$ ).
- ▶  $cl(X)$  is the set of *touching points* of  $X$  (the *closure* of  $X$ ).
- ▶  $bd(X)$  is the set of *boundary points* of  $X$ .
- ▶ A set is *closed* if  $X = cl(X)$ .

# Interior, Boundary, and Closure Operators

The function  $i(\cdot)$  is an **interior operator**:

1.  $i(\mathcal{U}) = \mathcal{U}$
2.  $i(X) \cap i(Y) = i(X \cap Y)$
3.  $i(X) \subseteq X$
4.  $i(i(X)) = i(X)$

**Note:**

- ▶  $X$  is **open** iff  $X = i(X)$
- ▶  $cl(X) = \overline{i(\overline{X})}$
- ▶  $bd(X) = cl(X) \cap cl(\overline{X})$

# From Interior Operators to Topologies and Back

## Theorem

Let  $\mathcal{U}$  be a set and  $i: 2^{\mathcal{U}} \rightarrow 2^{\mathcal{U}}$  be an “interior operator”. Define

$$\mathcal{O} := \{O \subseteq \mathcal{U} \mid O = i(O)\}.$$

Then  $\mathcal{T} = (\mathcal{U}, \mathcal{O})$  is a topological space.

## Proof.

Since  $i(\mathcal{U}) = \mathcal{U}$  by (1), we have  $\mathcal{U} \in \mathcal{O}$ . Since  $i(\emptyset) \subseteq \emptyset$  by (3), we have  $i(\emptyset) = \emptyset$ , and therefore  $\emptyset \in \mathcal{O}$ .

By (2),  $\mathcal{O}$  is closed under pairwise intersection. From (2), it follows that  $X \subseteq Y$  implies  $i(X) \subseteq i(Y)$  (which we need below).

Let  $O := \bigcup_{i \in I} O_i$ ,  $O_i = i(O_i)$  for all  $i$ . Of course,  $i(O) \subseteq O$ . Clearly,  $O_i \subseteq O$  for all  $i$ . Then  $O_i = i(O_i) \subseteq i(O)$ . Therefore  $O = \bigcup_{i \in I} O_i \subseteq i(O)$ . Hence,  $O = i(O)$ , i. e.,  $O \in \mathcal{O}$ . Thus,  $\mathcal{O}$  is closed under arbitrary unions. □

# Topological Set Expressions and Their Interpretations

**Topological set expressions** describe subsets of a topological space:

$$s \longrightarrow X \mid \top \mid \perp \mid s' \sqcap s'' \mid s' \sqcup s'' \mid \bar{s} \mid \mathbb{I}s',$$

with set variables  $X, Y, Z$ .

## Definition

A **topological interpretation** is a tuple  $I = (\mathcal{T}, d)$ , where  $\mathcal{T} = (\mathcal{U}, \mathcal{O})$  is a topological space with an associated **interior operator**  $i$  and  $d$  is a function from set variables to subsets of  $\mathcal{U}$ .

$d$  is extended to **topological set expressions** as follows:

$$\begin{array}{ll} d(\perp) & = \emptyset & d(\top) & = \mathcal{U} \\ d(s \sqcap s') & = d(s) \cap d(s') & d(s \sqcup s') & = d(s) \cup d(s') \\ d(\bar{s}) & = \mathcal{U} - d(s) & d(\mathbb{I}s) & = i(d(s)) \end{array}$$

# Topological Set Constraints

Elementary set constraints:

$$s \doteq t \quad \text{or} \quad s \not\doteq t$$

Complex set constraints: combinations using  $\wedge$ ,  $\vee$ , and  $\neg$ .

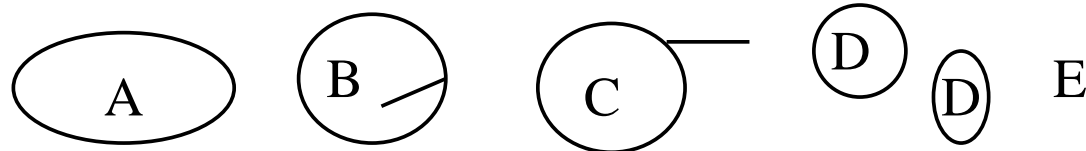
A topological interpretation  $I = (\mathcal{T}, d)$  satisfies a constraint:

$$I \models s \doteq t \quad \text{iff} \quad d(s) = d(t)$$

$$I \models s \not\doteq t \quad \text{iff} \quad d(s) \neq d(t)$$

As usual: model, satisfiability, equivalence, entailment, ...

# What Kind of Regions Do We Want to Consider?



A and D are **reasonable** regions, B, C, and E are not

In other words,  $X$  is a region iff it is **non-empty**

$$X \neq \perp$$

and **regular**, i. e., the closure of an open set:

$$X \doteq \overline{I(\overline{IX})}.$$

It is not necessary that a region is **internally connected**.

# Applying the Topological Set Constraints to RCC8

The **RCC8** relations are shorthands for topological set constraints:

$$\text{DC}(X, Y) := X \cap Y \doteq \perp$$

$$\text{EC}(X, Y) := X \cap Y \neq \perp \wedge \text{IX} \cap \text{IY} \doteq \perp$$

$$\text{PO}(X, Y) := \text{IX} \cap \text{IY} \neq \perp \wedge X \cap \bar{Y} \neq \perp \wedge \bar{X} \cap Y \neq \perp$$

$$\text{EQ}(X, Y) := X \doteq Y$$

$$\text{TPP}(X, Y) := X \cap \bar{Y} \doteq \perp \wedge X \cap \overline{\text{IY}} \neq \perp$$

$$\text{NTPP}(X, Y) := X \cap \overline{\text{IY}} \doteq \perp$$

In addition, each named region must satisfy **non-emptiness** and **regularity**.

~> It follows that the relations are **disjoint** and **exhaustive**.

# Normal Form Constraints

- ▶ A topological set constraint is in **normal form** if it is  $s \doteq \top$  or  $s \not\doteq \top$ .
- ▶ Every set constraint can be translated into normal form.
- ▶  $s \doteq t$  is equivalent to  $(\bar{s} \sqcup t) \sqcap (\bar{t} \sqcup s) \doteq \top$ 
  - ▶  $DC(X, Y) = \bar{X} \sqcup \bar{Y} \doteq \top$
  - ▶  $EC(X, Y) = \bar{X} \sqcup \bar{Y} \not\doteq \top \wedge \overline{IX} \sqcup \overline{IY} \doteq \top$
  - ▶ ...

## A Deduction Theorem

**Notation:**  $s \sqsubseteq t$  stands for  $\bar{s} \sqcup t \doteq \top$ .

### Theorem (Deduction Theorem, Nutt 99)

*Let  $s, t$  be set expressions. Then*

$$s \doteq \top \models t \doteq \top \text{ iff } \models Is \sqsubseteq It.$$

### Theorem (Convexity)

*The conjunctive set constraint*

$$s_1 \doteq \top \wedge \dots \wedge s_m \doteq \top \wedge t_1 \not\doteq \top \wedge \dots \wedge t_n \not\doteq \top$$

*is satisfiable if and only if the following constraints are satisfiable for each  $j \in \{1, \dots, n\}$*

$$s_1 \doteq \top \wedge \dots \wedge s_m \doteq \top \wedge t_j \not\doteq \top.$$

**Proof idea.**

# Topology and Modal Logic (1)

The **modal logic S4** can be characterized by the following axiom schemata (with **I** instead of  $\Box$  as the modal box operator)

- ▶  $\mathbf{I}\top \leftrightarrow \top$  (valid in **all frames**)
- ▶  $\mathbf{I}\varphi \rightarrow \varphi$  (valid in **T-frames**, reflexivity)
- ▶  $\mathbf{I}\varphi \wedge \mathbf{I}\psi \leftrightarrow \mathbf{I}(\varphi \wedge \psi)$  (valid in **all frames**)
- ▶  $\mathbf{I}\mathbf{I}\varphi \leftrightarrow \mathbf{I}\varphi$  (valid in **T4-frames**, transitivity & reflexivity)

**Reminder:** Interior operator

- ▶  $i(\mathcal{U}) = \mathcal{U}$
- ▶  $i(X) \subseteq X$
- ▶  $i(X) \cap i(Y) = i(X \cap Y)$
- ▶  $i(i(X)) = i(X)$

## Topology and Modal Logic (2)

Define a translation function  $\pi$  from **set expressions** to **S4 formulae** as follows:

- ▶  $\pi(X) = X$
- ▶  $\pi(\bar{s}) = \neg\pi(s)$
- ▶  $\pi(s \sqcap t) = \pi(s) \wedge \pi(t)$
- ▶  $\pi(s \sqcup t) = \pi(s) \vee \pi(t)$
- ▶  $\pi(\mathbf{I}s) = \mathbf{I}\pi(s)$

A set expression  $s$  is called a **topological tautology** if  $d(s) = \mathcal{U}$  for all topological interpretations  $I = ((\mathcal{U}, \mathcal{O}), d)$ .

### Theorem (McKinsey & Tarski 48)

*$s$  is a topological tautology iff  $\pi(s)$  is S4-valid.*

### Corollary

*$s$  is topologically satisfiable iff  $\pi(s)$  is S4-satisfiable.*

# Topological Set Constraints and Modal Logic (1)

How can we use this result for conjunctive topological set constraints?

1. Using the convexity theorem, we only have to test the satisfiability of constraints of the form

$$C_j = (s_1 \doteq \top \wedge \dots \wedge s_m \doteq \top \wedge t_j \not\doteq \top).$$

2.  $C_j$  is satisfiable iff  $s_1 \doteq \top \wedge \dots \wedge s_m \doteq \top \not\doteq t_j \doteq \top$ . Equivalently, we can test  $s \doteq \top \not\doteq t_j \doteq \top$ , with  $s = s_1 \sqcap \dots \sqcap s_m$ .
3. Using the deduction theorem, it suffices to check  $\not\doteq Is \sqsubseteq It_j$ , i. e., whether  $\overline{Is} \sqcup It_j$  is not a tautology, i. e., whether  $Is \sqcap \overline{It_j}$  is satisfiable. Using the McKinsey-Tarski theorem, this amounts to test for S4-satisfiability of  $\pi(Is \sqcap \overline{It_j})$ .

## Topological Set Constraints and Modal Logic (2)

### Theorem (Translation)

*The formula*

$$s_1 \dot{=} \top \wedge \dots \wedge s_m \dot{=} \top \wedge t_1 \not\dot{=} \top \wedge \dots \wedge t_n \not\dot{=} \top$$

*is satisfiable if for each  $j \in \{1, \dots, n\}$ , the following formula is S4-satisfiable:*

$$\mathbf{I}\pi(s_1) \wedge \dots \wedge \mathbf{I}\pi(s_m) \wedge \neg \mathbf{I}\pi(t_j).$$

## Topological Set Constraints and Modal Logic (3)

Let  $\Box$  and  $\Diamond$  be K-modalities.

### Proposition

Let  $\varphi_1, \dots, \varphi_m, \psi_1, \dots, \psi_n$  be multi-modal formulae not containing the K-operators  $\Box$  and  $\Diamond$ . Then

$$\Box\varphi_1 \wedge \dots \wedge \Box\varphi_m \wedge \Diamond\psi_1 \wedge \dots \wedge \Diamond\psi_n$$

is satisfiable iff for all  $j \in \{1, \dots, n\}$  the formulae

$$\varphi_1 \wedge \dots \wedge \varphi_m \wedge \psi_j$$

are satisfiable.

### Proof idea.

Create from models satisfying the later formula a modal interpretation for the former formula. □

## New Translation

Use a multi-modal logic for the translation. Extend  $\pi$  as follows:

- ▶  $\pi(s \doteq \top) = \Box \mathbf{I} \pi(s)$
- ▶  $\pi(s \not\dot{=} \top) = \Diamond \neg \mathbf{I} \pi(s)$
- ▶  $\pi(C_1 \wedge C_2) = \pi(C_1) \wedge \pi(C_2)$
- ▶ ...

This leads to the following translation of RCC8 constraints:

- ▶  $\pi(\text{DC}(X, Y)) = \Box \mathbf{I} \neg (X \wedge Y)$
- ▶  $\pi(\text{EC}(X, Y)) = \Box \mathbf{I} \neg (\mathbf{I} X \wedge \mathbf{I} Y) \wedge \Diamond \neg \mathbf{I} \neg (X \wedge Y)$
- ▶ ...

### Theorem (Translation)

*Let  $C$  be an arbitrary topological set constraint. Then  $C$  is satisfiable iff  $\pi(C)$  is satisfiable.*

# Outlook

- ▶ We wanted to state qualitative relationships between spatial regions
  - ▶ **Semantics:** Topology
  - ▶ **Language for describing relations:** Topological set constraints
  - ▶ ... can be translated to modal logic (McKinsey & Tarski)
  - ▶ Combination can be handled with another modality
- ~> Reasoning in RCC8?
- ~> Complexity?

# Reasoning with RCC8

RCC8 and Topology

## Reasoning with RCC8

Reminder

Dimension

Upper Bounds

Lower Bound – Proving NP-Hardness

Constraint Reasoning

Some Empirical Results

Outlook & Open Problems

# Reminder

- ▶ **RCC8** is a relation calculus for expressing spatial/topological information.
  - ▶ **Topology** is the right mathematical theory to give meaning to the RCC8 relations.
  - ▶ **Topological set constraints** can be used to characterize the relations.
  - ▶ Validity of **topological set expressions** is equivalent to S4-validity of translations of these set expressions.
  - ▶ Using an **additional K-modality**, satisfiability of the topological set constraints can be tested
- ↪ Reduction of spatial reasoning problems to modal logic reasoning problems.

# What is the Role of the Dimension?

- ▶ Already mentioned: McKinsey & Tarski do not mention dimension at all.

~> It has been shown:

- ▶ If an RCC8 CSP is topologically satisfiable, then it is satisfiable in **2 dimensions**, provided we do **not** require regions to be **internally connected**.
- ▶ If we require regions to be **internally connected** in **2 dimensions**, then the problem is open.
- ▶ If we allow **3 dimensions**, then the issue whether regions are internally connected is not crucial anymore.

## Upper Bounds – Using Results From Modal Logic

- ▶ Satisfiability in most modal logics (incl. K and S4 and multi-modal logics using these modalities) is **PSPACE-complete**.
- ▶ Upper bound from tableaux proofs: It suffices to explore one branch at a time in a depth-first manner and the depth is bounded polynomially by the size of the formula.
- ▶ Lower bound from reduction from QBF (quantified boolean formula).
- ▶ Deciding the satisfiability of **topological set constraints** is in PSPACE.

## A Better Upper Bound for RCC8

- ▶ Since the topological set constraints (and hence the modal formulae) resulting from RCC8 constraints are **very restricted**, there might be hope that we can do better than PSPACE.
  - ▶ Consider the nesting depth of **I** in modal formulae resulting from RCC8-formulae.
  - ▶ Satisfiability of S4-formulae with a fixed nesting depth is **NP-complete**.
- ↪ Guess a base relation (for each non-base relation) and then guess a satisfying interpretation.

### Proposition

*RCC8 satisfiability is in NP.*

# Constraint Propagation

- ▶ As in Allen's interval algebra, we may want to use **constraint propagation** instead of translating everything to modal logic.
- ▶ We need a composition table . . .
- ▶ . . . which could be computed using the modal logic encoding (and in fact, this has been done).
- ▶ Based on this table, we can then apply the path-consistency algorithm
- ▶ . . . and ask ourselves for which fragment of RCC8 it is **complete**.

# Composition Table

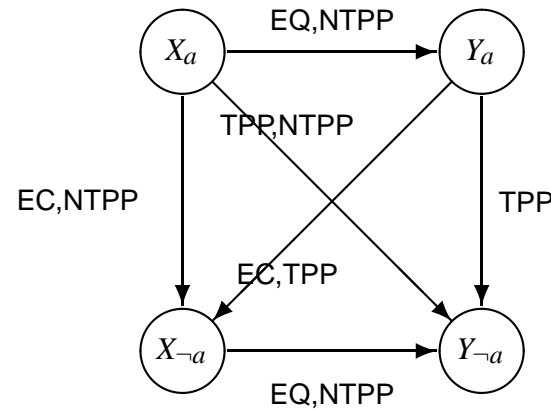
o	DC	EC	PO	TPP	NTPP	TPP <sup>-1</sup>	NTPP <sup>-1</sup>	EQ
DC	*	DC,EC PO,TPP NTPP	DC,EC PO,TPP NTPP	DC,EC PO,TPP NTPP	DC,EC PO,TPP NTPP	DC	DC	DC
EC	DC,EC PO,TPP <sup>-1</sup> NTPP <sup>-1</sup>	DC,EC PO,TPP TPP <sup>-1</sup> ,EQ	DC,EC PO,TPP NTPP	EC,PO TPP NTPP	PO TPP NTPP	DC,EC	DC	EC
PO	DC,EC PO,TPP <sup>-1</sup> NTPP <sup>-1</sup>	DC,EC PO,TPP <sup>-1</sup> NTPP <sup>-1</sup>	*	PO TPP NTPP	PO TPP NTPP	DC,EC PO, TPP <sup>-1</sup> NTPP <sup>-1</sup>	DC,EC PO,TPP <sup>-1</sup> NTPP <sup>-1</sup>	PO
TPP	DC	DC,EC	DC,EC PO,TPP NTPP	TPP NTPP	NTPP	DC,EC PO,TPP TPP <sup>-1</sup> ,EQ	DC,EC PO,TPP <sup>-1</sup> NTPP <sup>-1</sup>	TPP
NTPP	DC	DC	DC,EC PO,TPP NTPP	NTPP	NTPP	DC,EC PO,TPP NTPP	*	NTPP
TPP <sup>-1</sup>	DC,EC PO,TPP <sup>-1</sup> NTPP <sup>-1</sup>	EC,PO TPP <sup>-1</sup> NTPP <sup>-1</sup>	PO TPP <sup>-1</sup> NTPP <sup>-1</sup>	PO,EQ TPP TPP <sup>-1</sup>	PO TPP NTPP	TPP <sup>-1</sup> NTPP <sup>-1</sup>	NTPP <sup>-1</sup>	TPP <sup>-1</sup>
NTPP <sup>-1</sup>	DC,EC PO,TPP <sup>-1</sup> NTPP <sup>-1</sup>	PO TPP <sup>-1</sup> NTPP <sup>-1</sup>	PO TPP <sup>-1</sup> NTPP <sup>-1</sup>	PO TPP <sup>-1</sup> NTPP <sup>-1</sup>	PO,TPP <sup>-1</sup> TPP,NTPP NTPP <sup>-1</sup> ,EQ	NTPP <sup>-1</sup>	NTPP <sup>-1</sup>	NTPP <sup>-1</sup>
EQ	DC	EC	PO	TPP	NTPP	TPP <sup>-1</sup>	NTPP <sup>-1</sup>	EQ

# Lower Bound: Proving NP-Hardness

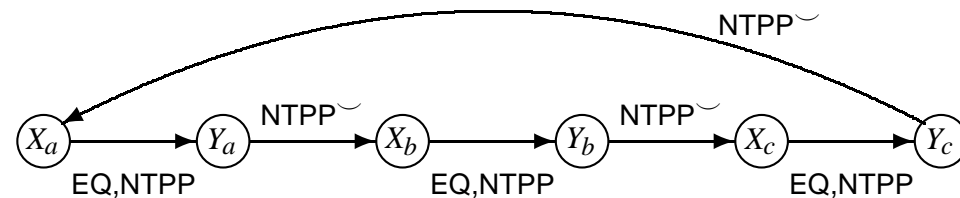
- ▶ **Idea:** Reduction from 3-SAT
- ▶ **3-SAT** structure
  1. Literals  $a, b, c$ : can be true or false
  2. Complementary literals:  $a$  is true iff  $\neg a$  is false
  3. Clauses  $l_1 \vee l_2 \vee l_3$ : at least one literal must be true
- ▶ **RCC8-CSP**
  1. **Truth value constraints**  $X_a \{R_t, R_f\} Y_a$ : Either  $X_a \{R_t\} Y_a$  or  $X_a \{R_f\} Y_a$  holds
  2. **Polarity constraints:**  $X_a \{R_t\} Y_a$  holds iff  $X_{\neg a} \{R_f\} Y_{\neg a}$  holds
  3. **Clause constraints:** At least one of  $X_{l_1} \{R_t\} Y_{l_1}$ ,  $X_{l_2} \{R_t\} Y_{l_2}$ , or  $X_{l_3} \{R_t\} Y_{l_3}$  holds

# The Reduction

- ▶ Relations:  $R_t = NTPP$ ,  $R_f = EQ$
- ▶ Polarity constraints:



- ▶ Clause constraints:



- ▶ RCC8 sat.  $\Rightarrow$  3-SAT: follows from reduction
- ▶ 3-SAT  $\Rightarrow$  RCC8 sat.: Construction of model for  $\Theta_\phi$  for each positive 3-SAT instance  $\phi$

# Tractable Fragments?

- ▶ As in the case of Allen's interval calculus, we may ask for **maximal tractable subsets** . . .
- ▶ Again, one can identify relations that can be encoded by **Horn formulae**.
- ▶ **Idea**: Consider relations that can be expressed in a way such that we have to consider only Horn formulae inside all worlds.
- ▶ **Idea**: Try to restrict the number of worlds to consider to a poly. number
- ▶ 148 **Horn relations**  $\mathcal{H}_8$ , which forms again a **maximal subset**.
- ▶ **Path consistency** is refutation complete for  $\mathcal{H}_8$ .
- ▶ There are **2 additional maximal subsets** that allow for poly. satisfiability testing!

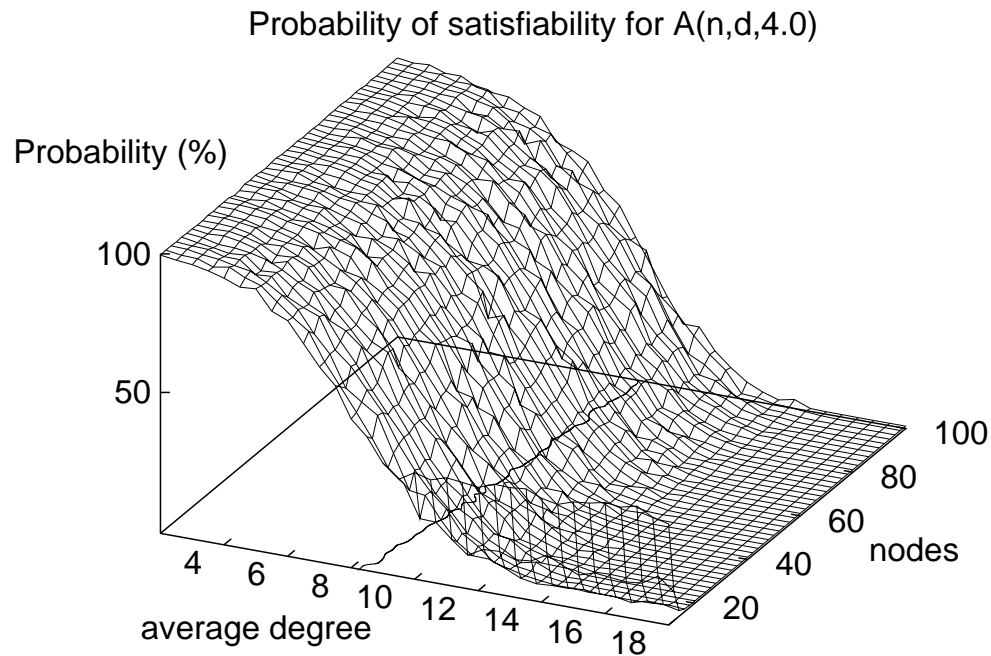
# Some Experiments

- ▶ How difficult is the RCC8 satisfiability problem **in practice**?
- ▶ Are there particularly **difficult instances**?
  - ↪ Where is the **phase transition** region?
  - ↪ Cheeseman et al [IJCAI 91] conjectured that for all NP-complete problems there exists a parameter such that when changing this parameter there exists a very small range – the **phase transition region** – where the probability of satisfiability of **randomly generated** instances changes from 1 to 0. They also conjectured that in this area one finds many **hard** instances.
- ▶ How well does the path consistency method **approximate** satisfiability?
- ▶ Can  $\mathcal{H}_8$  be used to speed up the satisfiability testing?

# Generating Instances

- ▶ Randomly generating instances according to the following **parameters**:
  - ▶ **Number of nodes**  $n$
  - ▶ Average **number of constraints**  $d$ :  $(nd/2$  out of  $n(n-1)/2$  possible constraints
  - ▶ Average **number of base relations**  $l$  per constraint
  - ▶ Allowed constraints
    - ▶  $A(n, d, l)$ : all RCC8 relations
    - ▶  $H(n, d, l)$ : only relations out of RCC8 –  $\mathcal{H}_8$

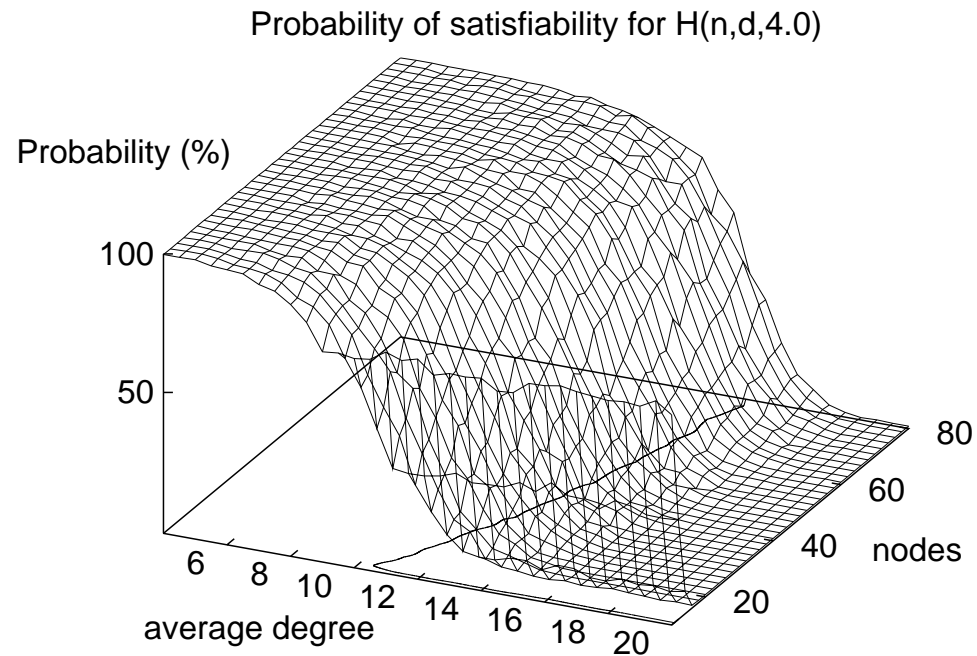
# Phase Transition for $A(n, d, 4)$



500 instances per data point

- **Phase transition** for  $A(n, d, 4)$  between  $d = 8$  and  $d = 10$  for  $10 \leq n \leq 100$ .

# Phase Transition for $H(n, d, 4)$



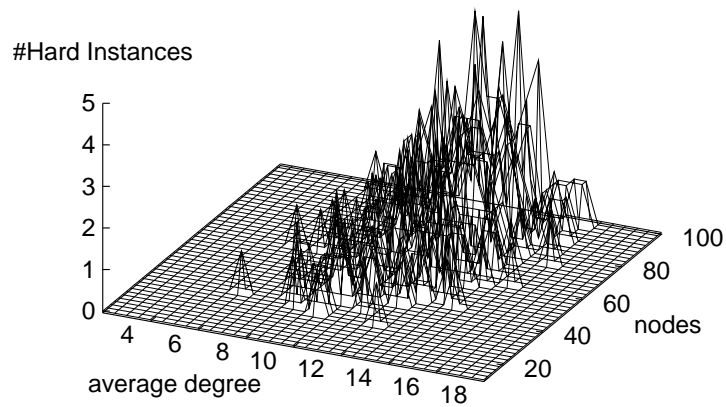
500 instances per data point

- **Phase transition** for  $H(n, d, 4)$  between  $d = 10$  and  $d = 15$  for  $10 \leq n \leq 80$ .

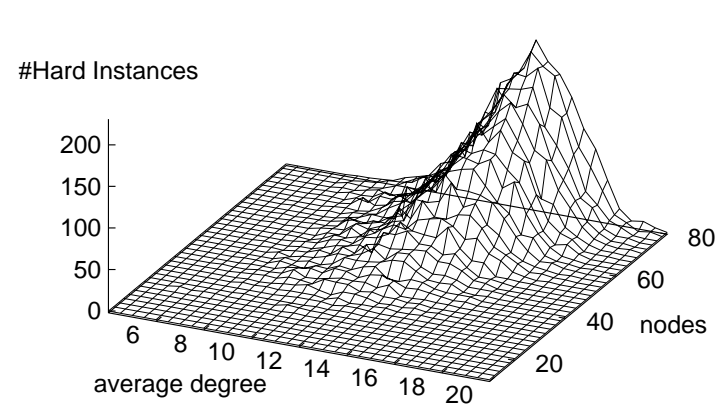
# Hard Instances ...

... using more than 10,000 search nodes

Number of hard instances for  $A(n,d,4.0)$



Number of hard instances for  $H(n,d,4.0)$



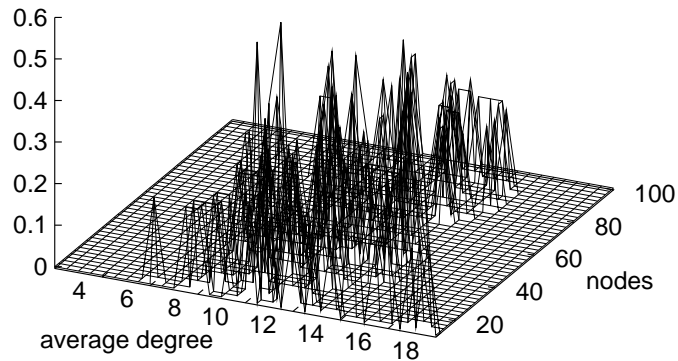
500 instances per data point

# Quality of Path Consistency...

... measured as the percentage of path consistent but unsatisfiable CSPs

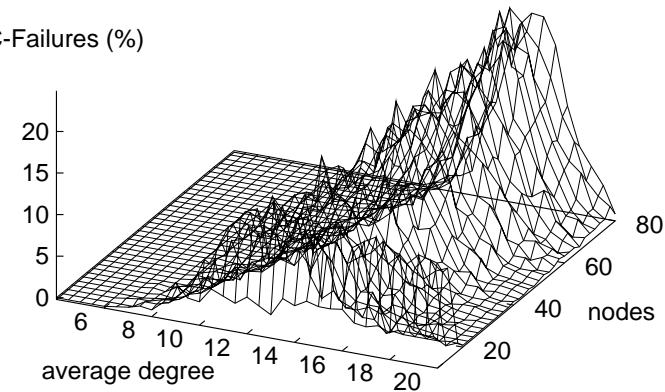
Percentage points of incorrect PCA answers for  $A(n,d,4.0)$

PC-Failures (%)



Percentage points of incorrect PCA answers for  $H(n,d,4.0)$

PC-Failures (%)



500 instances per data point

# Outlook & Open Problems

- ▶ RCC8 is the spatial-topological counterpart of **Allen's interval calculus**.
- ▶ Formalization can be done using **topology** and – because of McKinsey & Tarski's result – **modal logic**.
- ▶ Computationally **well behaved**.
- ▶ In contrast to Allen's calculus no applications so far.
- ▶ **Combinations** of RCC8 with other constraint spatial calculi.
- ▶ **Combining** RCC8 and Allen's interval calculus to form a **temporal-spatial calculus**.
- ▶ Are there other interesting spatial calculi?

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