Belief Revision

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Conclusion

- Changing our beliefs as the world or our information about it changes is belief revision.
- We start with some belief state K (a deductively closed set of formulae). When new information arrives, we change the belief state in order to *accommodate the new information*.
- If the new information contradicts the old beliefs, the new belief state is not (monotonically) a superset of the old belief state.
- Contrary to nonmonotonic reasoning, here we deal with temporal nonmonotonicity, i.e., the nonmonotonic evolution of a knowledge base or belief state over time.

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Two Scenarios: Update and Revision

- We have a theory about the world, and the new information is meant to *correct* our theory.

$\rightsquigarrow$ belief revision: change your belief state minimally in order to accommodate the new information.

- We have a correct theory about the current state of the world, and the new information is meant to record a *change* in the world.

$\rightsquigarrow$ belief update: incorporate the change by assuming that the world has changed minimally.

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Update and Revision are Different

Assume the new information is consistent with our old beliefs.

- In case of revision, we would like to add the new information monotonically to our old beliefs.
- For belief update this is not necessarily the case.
  - We know that the *door is open or the window is open*.
  - We learn that the world has changed and the *door is now closed*.
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Belief Change Operations

- General assumption: A belief state is modeled by a deductively closed theory K, i.e., $K = Th(K)$.
- $\mathcal{L}$: Logical language (propositional logic)
- $Th_{\mathcal{L}}$: Set of deductively closed theories (or belief sets) over $\mathcal{L}$
- Belief Change Operations:

$$\text{Monotonic addition:} \quad +: \mathrm{Th}_{\mathcal{L}} \times \mathcal{L} \rightarrow \mathrm{Th}_{\mathcal{L}}$$

$$K + \psi = \mathrm{Th}(K \cup \{\psi\})$$

$$\text{Revision:} \quad \dot{+}: \mathrm{Th}_{\mathcal{L}} \times \mathcal{L} \rightarrow \mathrm{Th}_{\mathcal{L}}$$

- *Reasonable* revision operations?
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($\dot{+}$2) $\varphi \in K \dot{+} \varphi$

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Canonical Revision Operations?

- The postulates constrain the space of revision operations, but do not choose one uniquely.
- Revision operations are closed under intersection, so should we choose the minimum?
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Examples

Partial Meet Revision (AGM): Preference information is given by a *selection function* γ over the set of maximal consistent subtheories $(K \downarrow \neg\varphi)$ of K that do not include $\neg\varphi$:

$$K \dotplus \varphi \stackrel{\text{def}}{=} \left(\bigcap \gamma(K \downarrow \neg\varphi)\right) + \varphi.$$

Cut Revision (GM): Preference information is given by complete preorder $\preceq$ over all $\psi \in K$:

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Revision – Viewed Computationally

- We don't want to deal with deductively closed (infinite) theories.
- Consider belief bases (finite sets of formulae) as *representing* belief sets.
- A theory K over the propositional logic $\mathcal{L}$ with n propositional atoms can have as many as
  - 2^{2^n} different propositions and
  - 2^n different models.
- Consider ways of specifying preference information in a *concise* way, i.e., polynomial in the size of the belief base.

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Base Revision

- We start from a finite belief base A and preference information over the elements of A.
- We want to generate a revision operation (restricted to Th(A)).
- Assume a partitioning of A into n *priority classes* $A_1, \ldots A_n$ such that the elements of A_i are more important/preferred than those of A_j for $i > j$.
- Define a revision operation that keeps as many of the more important formulae as possible.

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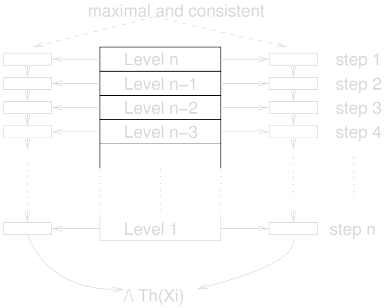
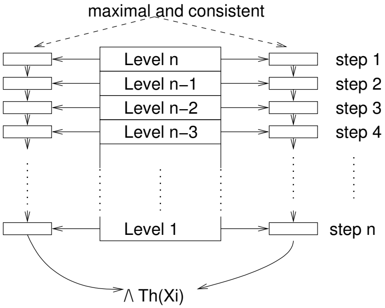
Prioritized Meet Base Revision (PMBR)

Prioritized Meet Base Revision is defined by maximizing the most important formulae.



Prioritized Meet Base Revision (PMBR)

Prioritized Meet Base Revision is defined by maximizing the most important formulae.



Prioritized Meet Base Revision (PMBR) – Formally

Definition ($\Downarrow$)

Let A be a base with partitioning $A_1, \ldots, A_n$.
$(A \Downarrow \varphi)$ consists of sets $S = S_1 \cup \cdots \cup S_n$ fulfilling the following for all $i \in \{n, \ldots, 1\}$.

$S_{n+1} = \emptyset$

S_i is a maximal subset of $A_i \cup S_{i+1}$ such that $S_{i+1} \subseteq S_i$ and $S_i \not\models \varphi$

Prioritized Meet Base Revision – Example of $\Downarrow$

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Compute one element of $A \Downarrow \neg F$.

$$\begin{array}{lll} A_4 & = & \{P \rightarrow B, B \rightarrow F\} \\ A_3 & = & \{P \rightarrow \neg F\} \\ A_2 & = & \{B\} \\ A_1 & = & \{P\} \end{array} \qquad \begin{array}{lll} S_4 & = & \{P \rightarrow B, B \rightarrow F\} \\ S_3 & = & \{P \rightarrow \neg F\} \cup S_4 \\ S_2 & = & \{B\} \cup S_3 \\ S_1 & = & \{\} \cup S_2 \end{array}$$

Actually, $S_1 \cup S_2 \cup S_3 \cup S_4$ is the only element of

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Prioritized Meet Base Revision (PMBR) – Formally

Definition

$$A \oplus \varphi \stackrel{\text{def}}{=} \Big(\bigcap_{B \in (A \Downarrow \neg\varphi)} \mathsf{Th}(B) \Big) + \varphi.$$

Definition

Revision operation $\dot{+}$ on $\mathsf{Th}(A)$ (that depends on A and the priority information) is

$$\mathsf{Th}(A) \dot{+} \varphi \stackrel{\text{def}}{=} A \oplus \varphi.$$

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Properties of PMBRs

- Generates *partial meet revision*, but does not satisfy $(\dot{+}8)$ in general.
- Deciding whether $A \oplus \varphi \models \psi$ is Π_2^p-complete, even for one priority class. (Proof is similar to Π_2^p-completeness of PDS for default logic.)
- A revised base can be represented by

$$A \oplus \varphi \quad = \quad \text{Th}\Big(\big(\bigvee(A \Downarrow \neg\varphi)\big) \wedge \varphi\Big).$$

- A revised base can become exponentially large:

$$A \quad = \quad \{p_1, \ldots, p_m, q_1, \ldots, q_m\}$$

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Cut Base Revision (CBR)

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Let $\widehat{A_j} \stackrel{\text{def}}{=} \bigcup_{i=j}^{n} A_i$, then cut base revision $\otimes$ is defined as:

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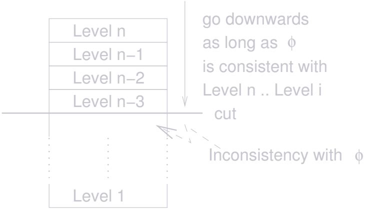

- *Easy* to compute: in $\text{P}^{\text{NP}[O(\log n)]}$.
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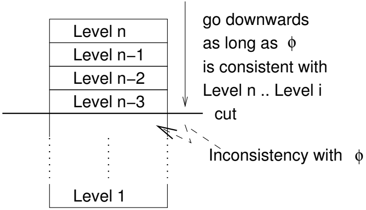

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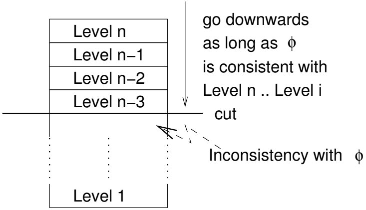

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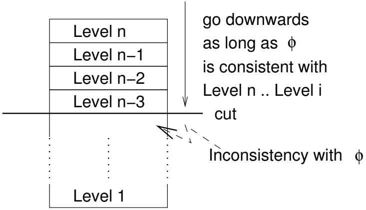

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Less conservative Linear Base Revision (LBR)

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Definition (Idea informally)

Throw away an entire priority class only if it would lead to a *contradiction* which cannot be blamed on a lower classes $\rightsquigarrow$ *linear base-revision* $\odot$.

- Generates revision operations satisfying the AGM postulates.
- Complexity: Δ_2^p-complete; $O(n^2)$ for Horn logic.
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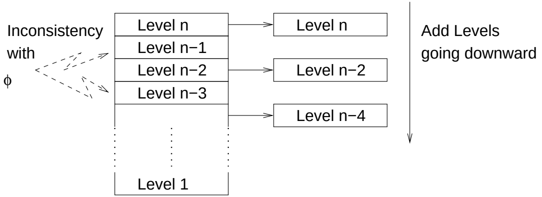

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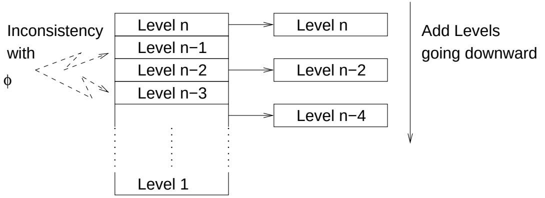

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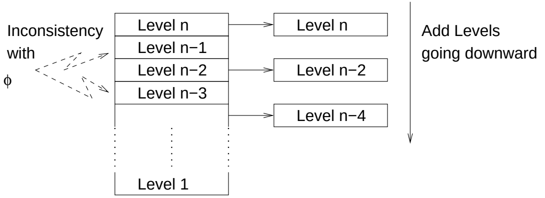

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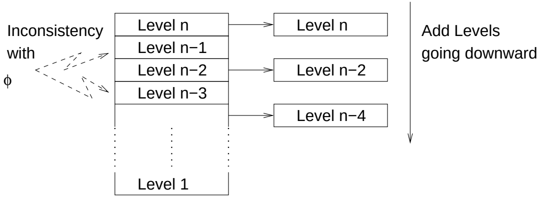

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Revision vs. Nonmonotonic Reasoning

Belief Revision and Nonmonotonic Reasoning seem to be of different nature, but there is a tight connection:

- Given K and a revision operation $\dot{+}$

$\rightsquigarrow$ a nonmonotonic consequence relation can be defined as follows: $\phi \mid\!\sim \psi$ iff $\psi \in K \dot{+} \phi$.

In this case,

- the rationality postulates correspond to principles of NMR (such as cautious monotony etc.);
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NMR Principles and Rationality Postulates

(∔2) $\varphi \in K \dotplus \varphi$;

- *Reflexivity*

(∔3) $K \dotplus \varphi \subseteq K + \varphi$;

- *Supraclassicality*

(∔6) If $\models \varphi \leftrightarrow \psi$ then $K \dotplus \varphi = K \dotplus \psi$;

- *Left Logical Equivalence*

(∔8) If $\neg\psi \notin K \dotplus \varphi$,
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Outlook & Summary

- While NMR and belief revision seem to be two sides of the same coin, there are notable *pragmatic differences*:
  - Belief revision seems to require that we can easily represent the changed belief base, while for NMR it makes sense to use dense representations.
  - A similar argument could be made for the computational complexity.
- NMR and Belief Revision can be thought of as *qualitative ways* of dealing with uncertainty in a purely logical setting.
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