

# Principles of Knowledge Representation and Reasoning

## Nonmonotonic Reasoning

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## A Motivating Example: Defaults in Knowledge Bases

1. `employee(anne)`
2. `employee(bert)`
3. `employee(carla)`
4. `employee(detlef)`
5. `employee(thomas)`
6. `onUnpaidMPaternityLeave(thomas)`
7.  $\text{employee}(X) \wedge \neg \text{onUnpaidMPaternityLeave}(X) \rightarrow \text{gettingSalary}(X)$
8. **typically:**  $\text{employee}(X) \rightarrow \neg \text{onUnpaidMPaternityLeave}(X)$

# A Motivating Example: Common Sense Reasoning

1. Tweety is a bird like other birds.
2. During the summer he stays in Northern Europe, in the winter he stays in Africa.
  - ▶ Would you expect Tweety to be able to fly?
  - ▶ How does Tweety get from Northern Europe to Africa?

How would you formalize this in formal logic so that you get the expected answers?

## A Formalization ...

1.  $\text{bird}(\text{tweety})$
  2.  $\text{spend-summer}(\text{tweety}, \text{northern-europe}) \wedge \text{spend-winter}(\text{tweety}, \text{africa})$
  3.  $\forall x(\text{bird}(x) \rightarrow \text{can-fly}(x))$
  4.  $\text{far-away}(\text{northern-europe}, \text{africa})$
  5.  $\forall xyz(\text{can-fly}(x) \wedge \text{far-away}(y, z) \wedge \text{spend-summer}(x, y) \wedge \text{spend-winter}(x, z) \rightarrow \text{flies}(x, y, z))$
- ▶ The implication (3) is just a **reasonable assumption**
  - ▶ What if Tweety is an **emu**?

## Examples of Such Reasoning Patterns

- Closed world assumption:** Data-base of **ground atoms**. All ground atoms not present are **assumed** to be false.
- Negation as failure:** In PROLOG, **NOT(P)** means “P is not **provable**” instead of “P is provably false”.
- Non-strict inheritance:** An attribute value is **inherited** only if there is no more specialized information contradicting the attribute value.
- Reasoning about actions:** When reasoning about actions, it is usually assumed that a property **changes** only if it **has to change**, i.e., properties by default do not change.

# Default, Defeasible, and Non-monotonic Reasoning

**Default Reasoning:** Jump to a conclusion if there is no information that contradicts the conclusion.

**Defeasible Reasoning:** Reasoning based on assumptions that can turn out to be wrong, — i.e., conclusions are defeasible. In particular, default reasoning is defeasible.

**Non-monotonic Reasoning:** In classical logic, the set of consequence grows monotonically with the set of premises. If reasoning is defeasible, then reasoning becomes non-monotonic.

# Approaches to Non-Monotonic Reasoning

- ▶ **Consistency-based:** Extend classical theory by rules that test whether an assumption is consistent with existing beliefs
- ⇒ non-monotonic logics like DL (default logic), NMLP (non-monotonic logic programming)
- ▶ **Entailment-based on normal models:** Models are ordered by normality. Entailment is determined by considering the most normal models only.
- ⇒ Circumscription, Preferential and Cumulative Logics

# NM Logic – Consistency-Based

If  $\varphi$  typically implies  $\psi$ ,  $\varphi$  is given, and it is consistent to assume  $\psi$ , then conclude  $\psi$ .

1. Typically  $\text{bird}(x)$  implies  $\text{can-fly}(x)$

2.  $\forall x(\text{emu}(x) \rightarrow \text{bird}(x))$

3.  $\forall x(\text{emu}(x) \rightarrow \neg \text{can-fly}(x))$

4.  $\text{bird}(\text{tweety})$

$\Rightarrow \text{can-fly}(\text{tweety})$

5. ... +  $\text{emu}(\text{tweety})$

$\Rightarrow \neg \text{can-fly}(\text{tweety})$

## NM Logic – Normal Models

If  $\varphi$  typically implies  $\psi$ , then the models satisfying  $\varphi \wedge \psi$  should be **more normal** than those satisfying  $\varphi \wedge \neg\psi$ .

Similarly, try to **minimize** the interpretation of “**Ab**normality” predicates.

1.  $\forall x(\text{bird}(x) \wedge \neg\text{Ab}(x) \rightarrow \text{can-fly}(x))$
2.  $\forall x(\text{emu}(x) \rightarrow \text{bird}(x))$
3.  $\forall x(\text{emu}(x) \rightarrow \neg\text{can-fly}(x))$
4.  $\text{bird}(\text{tweety})$

**Minimize interpretation** of **Ab**.

$\Rightarrow \text{can-fly}(\text{tweety})$

5. ... +  $\text{emu}(\text{tweety})$

$\Rightarrow$  Now in all models (incl. the normal ones):  $\neg \text{can-fly}(\text{tweety})$

# Default Logic – Outline

## Introduction

## Default Logic

- Basics

- Extensions

- Properties of Extensions

- Normal Defaults

- Default Proofs

- Decidability

- Propositional DL

## Complexity of Default Logic

## Literature

## Motivation: Reiter's Default Logic

- ▶ We want to express something like “typically birds fly”.
- ▶ Add **non-logical inference rule**

$$\frac{\text{bird}(x) : \text{can-fly}(x)}{\text{can-fly}(x)}$$

with the **intended meaning**:

*If  $x$  is a bird and if it is consistent to assume that  $x$  can fly, then conclude that  $x$  can fly.*

- ▶ **Exceptions** can be represented as formulae:

$$\begin{aligned} \forall x(\text{penguin}(x) \rightarrow \neg \text{can-fly}(x)) \\ \forall x(\text{emu}(x) \rightarrow \neg \text{can-fly}(x)) \\ \forall x(\text{kiwi}(x) \rightarrow \neg \text{can-fly}(x)) \end{aligned}$$

# Formal Framework

- ▶ FOL with classical provability relation  $\vdash$  and deductive closure:

$$\text{Th}(\Phi) := \{\phi \mid \Phi \models \phi\}$$

- ▶ Default rules:  $\frac{\alpha: \beta}{\gamma}$

$\alpha$ : Prerequisite: must have been derived before rule can be applied.

$\beta$ : Consistency condition: the negation may not be derivable.

$\gamma$ : Consequence: will be concluded.

- ▶ A default rule is **closed** if it does not contain free variables.
- ▶ **(Closed) default theory**: A pair  $(D, W)$ , where  $D$  is a countable set of (closed) default rules and  $W$  is a countable set of FOL formulae.

## Extensions of Default Theories

Default theories **extend** the theories given by  $W$  using the default rules  $D$  ( $\rightsquigarrow$  **extensions**). There may be zero, one, or many extensions.

### Example

$$W = \{a, \neg b \vee \neg c\}$$

$$D = \left\{ \frac{a: b}{b}, \frac{a: c}{c} \right\}$$

One **extension** contains  $b$ , the other contains  $c$ .

**Intuitively:** an **extension** is a set of **beliefs** resulting from  $W$  and  $D$ .

# Decision Problems about Extensions in Default Logic

**Existence of extensions:** Does a default theory have an extension?

**Credulous reasoning:** If  $\varphi$  is in at least one extension,  $\varphi$  is a **credulous default conclusion**.

**Skeptical Reasoning:** If  $\varphi$  is in all extensions,  $\varphi$  is a **skeptical default conclusion**.

## Extensions – Informally

Desirable properties of an **extension**  $E$  of  $(D, W)$ :

1. Contains all facts  $W \subseteq E$ .
2. Is deductively closed:  $E = \text{Th}(E)$ .
3. All applicable default rules have been applied:

**If**

$$3.1 \left( \frac{\alpha:\beta}{\gamma} \right) \in D,$$

$$3.2 \alpha \in E,$$

$$3.3 \neg\beta \notin E$$

**then**  $\gamma \in E$ .

$\Rightarrow$  Requirement: Application of default rules must follow in sequence (*groundedness*).

# Groundedness

## Example

$$W = \emptyset$$
$$D = \left\{ \frac{a: b}{b}, \frac{b: a}{a} \right\}$$

*Question:* Should  $\text{Th}(\{a, b\})$  be an extension?

*Answer:* No!

$a$  can only be derived if we already have derived  $b$ .

$b$  can only be derived if we already have derived  $a$ .

# Extensions – Formally

## Definition

Let  $\Delta = (D, W)$  be a closed default theory and let  $E$  be a set of closed formulae.

Let

$$E_0 = W$$

$$E_i = \text{Th}(E_{i-1}) \cup \left\{ \gamma \mid \frac{\alpha : \beta}{\gamma} \in D, \alpha \in E_{i-1}, \neg\beta \notin E \right\}$$

Then  $E$  is an **extension** of  $\Delta$  iff

$$E = \bigcup_{i=0}^{\infty} E_i.$$

## How to Use This Definition?

- ▶ The definition does not tell us how to **construct** an extension.
- ▶ However, it tells us how to **check** whether a set is an extension.
- ▶ Guess a set  $E$ .
- ▶ Then construct sets  $E_i$  by starting with  $W$ .
- ▶ If  $\boxed{E = \bigcup_{i=0}^{\infty} E_i}$ , then  $E$  is an **extension** of  $(D, W)$ .

## Examples

$$D = \left\{ \frac{a: b}{b}, \frac{b: a}{a} \right\}$$

$$W = \{a \vee b\}$$

$$D = \left\{ \frac{a: b}{\neg b} \right\}$$

$$W = \emptyset$$

$$D = \left\{ \frac{a: b}{\neg b} \right\}$$

$$W = \{a\}$$

$$D = \left\{ \frac{: a}{a}, \frac{: b}{b}, \frac{: c}{c} \right\}$$

$$W = \{b \rightarrow \neg a \wedge \neg c\}$$

$$D = \left\{ \frac{: c}{\neg d}, \frac{: d}{\neg e}, \frac{: e}{\neg f} \right\}$$

$$W = \emptyset$$

$$D = \left\{ \frac{: c}{\neg d}, \frac{: d}{\neg c} \right\}$$

$$W = \emptyset$$

$$D = \left\{ \frac{a: b}{c}, \frac{a: d}{e} \right\}$$

$$W = \{a, \neg b \vee \neg d\}$$

# Questions, Questions, Questions . . .

- ▶ What can we say about the **existence** of extensions?
- ▶ How are the different extensions **related** to each other?
  - ▶ Can one extension be a **subset** of another one?
  - ▶ Are extensions **pairwise incompatible** (i.e. jointly inconsistent)?
- ▶ Can an extension be **inconsistent**?

# Properties of Extensions

## Theorem

1. *If  $W$  is inconsistent, there is only one extension.*
2. *A closed default theory  $(D, W)$  has an inconsistent extension iff  $W$  is inconsistent.*

## Proof idea.

1. If  $W$  is inconsistent, no default rule is applicable and  $\text{Th}(W)$  is the only extension.
2. Claim 1  $\implies$  the *if*-part. For *only if*: If  $W$  is consistent, there is a consistent  $E_i$  s.t.  $E_{i+1}$  is inconsistent. Let  $\{\gamma_1, \dots, \gamma_n\} = E_{i+1} \setminus \text{Th}(E_i)$  (the conclusions of applied defaults). Now  $\{\neg\beta_1, \dots, \neg\beta_n\} \cap E = \emptyset$  because otherwise the defaults are not applicable.

But this contradicts the inconsistency of  $E$ .



# Properties of Extensions

## Theorem

If  $E$  and  $F$  are extensions of  $(D, W)$  such that  $E \subseteq F$ , then  $E = F$ .

## Proof sketch.

$E = \bigcup_{i=0}^{\infty} E_i$  and  $F = \bigcup_{i=0}^{\infty} F_i$ . Use induction to show  $F_i \subseteq E_i$ .

Base case  $i = 0$ : Trivially  $E_0 = F_0 = W$ .

Inductive case  $i \geq 1$ : Assume  $\gamma \in F_{i+1}$ . Two cases:

1.  $\gamma \in \text{Th}(F_i)$  implies  $\gamma \in \text{Th}(E_i)$  (because  $F_i \subseteq E_i$  by IH), and therefore  $\gamma \in E_{i+1}$ .
2. Otherwise  $\frac{\alpha:\beta}{\gamma} \in D$ ,  $\alpha \in F_i$ ,  $\neg\beta \notin F$ . However, then we have  $\alpha \in E_i$  (because  $F_i \subseteq E_i$ ) and  $\neg\beta \notin E$  (because of  $E \subseteq F$ ), i.e.,  $\gamma \in E_{i+1}$ .



## Normal Default Theories

All defaults in a **normal default theory** are **normal**:

$$\frac{\alpha : \beta}{\beta}.$$

### Theorem

*Normal default theories have at least one extension.*

### Proof sketch.

If  $W$  inconsistent, trivial. Otherwise construct

$$\begin{aligned} E_0 &= W \\ E_{i+1} &= \text{Th}(E_i) \cup T_i \end{aligned} \quad E = \bigcup_{i=0}^{\infty} E_i$$

where  $T_i$  is a maximal set s.t. (1)  $E_i \cup T_i$  is consistent and (2) if  $\beta \in T_i$  then there is  $\frac{\alpha : \beta}{\beta} \in D$  and  $\alpha \in E_i$ .

Show:  $T_i = \left\{ \beta \mid \frac{\alpha : \beta}{\beta} \in D, \alpha \in E_i, \neg \beta \notin E \right\}$  for all  $i \geq 0$ . □

# Normal Default Theories: Extensions are Orthogonal

## Theorem (Orthogonality)

*Let  $E$  and  $F$  be two extensions of a normal default theory. Then  $E \cup F$  is inconsistent.*

### Proof.

Let  $E = \bigcup E_i$  and  $F = \bigcup F_i$  with

$$E_{i+1} = \text{Th}(E_i) \cup \left\{ \beta \mid \frac{\alpha : \beta}{\beta} \in D, \alpha \in E_i, \neg\beta \notin E \right\}$$

and the same for  $F$ . Since  $E \neq F$ , there exists a smallest  $i$  such that  $E_{i+1} \neq F_{i+1}$ . This means there exists  $\frac{\alpha : \beta}{\beta} \in D$  with  $\alpha \in E_i = F_i$  but  $\beta \in E_{i+1}$  and  $\beta \notin F_{i+1}$ . This is only possible if  $\neg\beta \in F$ . This means  $\beta \in E$  and  $\neg\beta \in F$ , i.e.,  $E \cup F$  is inconsistent. □

# Default Proofs in Normal Default Theories

## Definition

A **default proof** of  $\gamma$  in a normal default theory  $(D, W)$  is a finite sequence of defaults  $(\delta_i = \frac{\alpha_i : \beta_i}{\beta_i})_{i=1, \dots, n}$  such that

1.  $W \cup \{\beta_1, \dots, \beta_n\} \vdash \gamma$ ,
2.  $W \cup \{\beta_1, \dots, \beta_n\}$  is consistent, and
3.  $W \cup \{\beta_1, \dots, \beta_k\} \vdash \alpha_{k+1}$ , for  $0 \leq k \leq n - 1$ .

## Theorem

Let  $\Delta = \langle D, W \rangle$  be a normal default theory so that  $W$  is consistent. Then  $\gamma$  has a default proof in  $\Delta$  iff there exists an extension  $E$  of  $\Delta$  such that  $\gamma \in E$ .

Test 2 (**consistency**) in the proof procedure suggests that default provability is not even **semi-decidable**.

# Decidability

## Theorem

*It is not semi-decidable to test whether a formula follows (skeptically or credulously) from a default theory.*

## Proof.

Let  $(D, W)$  be a default theory with  $W = \emptyset$  and  $D = \left\{ \frac{:\beta}{\beta} \right\}$  with  $\beta$  an arbitrary closed FOL formula. Clearly,  $\beta$  is in some/all extensions of  $(D, W)$  if and only if  $\beta$  is satisfiable.

The existence of a semi-decision procedure for default proofs implies that there is a semi-decision procedure for satisfiability in FOL.

But this is not possible because FOL validity is semi-decidable and this together with semi-decidability of FOL satisfiability would imply decidability of FOL, which is not the case. □

# Propositional Default Logic

- ▶ Propositional DL is decidable.
- ▶ How difficult is reasoning in propositional DL?
- ▶ The skeptical default reasoning problem (does  $\varphi$  follow from  $\Delta$  skeptically:  $\Delta \vdash \varphi$ ?) is called PDS, credulous reasoning is called LPDS.
- ▶ (L)PDS is co-NP-hard (let  $D = \emptyset$ ,  $W = \emptyset$ ) and NP-hard (let  $W = \emptyset$ ,  $D = \left\{ \frac{:\beta}{\beta} \right\}$ ).

# Complexity of DL – Outline

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Default Logic

Complexity of Default Logic

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- Semi-Normal Defaults

- Open Defaults

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# Skeptical Reasoning in Propositional DL

## Lemma

$PDS \in \Pi_2^P$ .

## Proof.

We show that the complementary problem **UNPDS** (is there an extension  $E$  such that  $\varphi \notin E$ ) is in  $\Sigma_2^P$ .

The **algorithm**: **Guess** set  $T \subseteq D$  of defaults: those that are applied.

**Verify** that defaults in  $T$  lead to  $E$ , using a **SAT oracle** and the guessed  $E = \text{Th} \left( \left\{ \gamma \mid \frac{\alpha:\beta}{\gamma} \in T \right\} \cup W \right)$ .

**Verify** that  $\left\{ \gamma \mid \frac{\alpha:\beta}{\gamma} \in T \right\} \cup W \not\models \varphi$  (**SAT oracle**).

$\rightsquigarrow$  UNPDS  $\in \Sigma_2^P$ . □

**Note**: LPDS  $\in \Sigma_2^P$ .

## $\Pi_2^P$ -Hardness

### Lemma

*PDS is  $\Pi_2^P$ -hard.*

### Proof.

Reduction from 2QBF to UNPDS: For  $\exists \vec{a} \forall \vec{b} \phi(\vec{a}, \vec{b})$  with  $\vec{a} = a_1, \dots, a_n$  and  $\vec{b} = b_1, \dots, b_m$  construct  $\Delta = (D, W)$  with

$$D = \left\{ \frac{:a_i}{a_i}, \frac{:\neg a_i}{\neg a_i}, \frac{:\neg \phi(\vec{a}, \vec{b})}{\neg \phi(\vec{a}, \vec{b})} \right\}, \quad W = \emptyset$$

No extension contains both  $a_i$  and  $\neg a_i$ .

Now

$\Delta \not\models \neg \phi(\vec{a}, \vec{b})$  iff there is extension  $E$  s.t.  $\neg \phi(\vec{a}, \vec{b}) \notin E$   
 iff there is  $E$  s.t.  $\phi(\vec{a}, \vec{b}) \in E$  (by  $\frac{:\neg \phi(\vec{a}, \vec{b})}{\neg \phi(\vec{a}, \vec{b})} \in D$ )  
 iff there is  $A \subset \{a_1, \neg a_1, \dots, a_n, \neg a_n\}$  s.t.  $A \models \phi(\vec{a}, \vec{b})$   
 iff  $\exists \vec{a} \forall \vec{b} \phi(\vec{a}, \vec{b})$  is true. □

## Conclusions & Remarks

### Theorem

*PDS is  $\Pi_2^P$ -complete, even for defaults of the form  $\frac{:\alpha}{\alpha}$ .*

### Theorem

*LPDS is  $\Sigma_2^P$ -complete, even for defaults of the form  $\frac{:\alpha}{\alpha}$ .*

- ▶ PDS is “easier” than reasoning in most modal logics.
- ▶ General and normal defaults have the same complexity.
- ▶ Polynomial special cases cannot be achieved by restricting, for example, to **Horn clauses** (satisfiability testing in polynomial time).
- ▶ It is necessary to restrict the underlying **monotonic reasoning problem** and the **number of extensions**.
- ▶ Similar results hold for other **non-monotonic logics**.

## Semi-Normal Defaults (1)

Semi-normal defaults are sometimes useful:

$$\frac{\alpha : \beta \wedge \gamma}{\beta}$$

Important when one has **interacting** defaults:

$$\frac{\text{Adult}(x) : \text{Employed}(x)}{\text{Employed}(x)}$$

$$\frac{\text{Student}(x) : \text{Adult}(x)}{\text{Adult}(x)}$$

$$\frac{\text{Student}(x) : \neg\text{Employed}(x)}{\neg\text{Employed}(x)}$$

For **Student(TOM)** we get two extensions: one with  $\text{Employed}(\text{Tom})$  and the other one with  $\neg\text{Employed}(\text{Tom})$ .

Since the third rule is “**more specific**”, we may prefer it.

## Semi-Normal Defaults (2)

- ▶ Since being a student is an exception, we could use a **semi-normal** default to exclude students from employed adults:

$$\frac{\text{Student}(x) : \neg\text{Employed}(x)}{\neg\text{Employed}(x)}$$

$$\frac{\text{Adult}(x) : \text{Employed}(x) \wedge \neg\text{Student}(x)}{\text{Employed}(x)}$$

$$\frac{\text{Student}(x) : \text{Adult}(x)}{\text{Adult}(x)}$$

- ▶ Representing conflict-resolution by semi-normal defaults becomes clumsy when the number of default rules becomes high.
- ▶ A scheme for assigning **priorities** would be more elegant (there are indeed such schemes).

# Open Defaults (1)

- ▶ Our examples included **open defaults**, but the theory covers only **closed defaults**.
- ▶ If we have  $\frac{\alpha(\vec{x}):\beta(\vec{x})}{\gamma(\vec{x})}$ , then the variables should stand for all *nameable* objects.
- ▶ **Problem**: What about objects that have been introduced implicitly:  
 $\boxed{\exists xP(x)}$ .
- ▶ **Solution by Reiter**: Skolemization of all formulae in  $W$  and  $D$ .
- ▶ **Interpretation**: An open default stands for all the closed defaults resulting from substituting **ground terms** for the variables.

## Open Defaults (2)

Skolemization can create problems because it preserves satisfiability, but it is not an equivalence transformation.

### Example

$$\forall x(\text{Man}(x) \leftrightarrow \neg \text{Woman}(x))$$

$$\forall x(\text{Man}(x) \rightarrow (\exists y(\text{Spouse}(x, y) \wedge \text{Woman}(y)) \vee \text{Bachelor}(x)))$$

$$\text{Man}(\text{TOM})$$

$$\text{Spouse}(\text{TOM}, \text{MARY})$$

$$\text{Woman}(\text{MARY})$$

$$\frac{: \text{Man}(x)}{\text{Man}(x)}$$

Skolemization of  $\exists y$ : ... enables concluding **Bachelor(TOM)**!

The reason is that for  $g(\text{TOM})$  we get  $\text{Man}(g(\text{TOM}))$  **by default** ( $g$  is the Skolem function).

## Open Defaults (3)

It is even worse: Logically equivalent theories can have different extensions.

$$\begin{aligned}
 W_1 &= \{\exists x(P(C, x) \vee Q(C, x))\} \\
 W_2 &= \{\exists xP(C, x) \vee \exists xQ(C, x)\} \\
 D &= \left\{ \frac{P(x, y) \vee Q(x, y): R}{R} \right\}
 \end{aligned}$$

$W_1$  and  $W_2$  are logically equivalent. However, the Skolemization of  $W_1$ , symbolically  $s(W_1)$ , is not equivalent with  $s(W_2)$ . The only extension of  $(D, W_1)$  is  $\text{Th}(s(W_1) \cup R)$ . The only extension of  $(D, W_2)$  is  $\text{Th}(s(W_2))$ .

**Note:** Skolemization is not the right method to deal with open defaults in the general case.

# Outlook

Although Reiter's definition of DL makes sense, one can come up with a number of variations and extend the investigation ...

- ▶ Extensions can be defined differently (e.g., by remembering consistency conditions).
- ▶ ... or by removing the groundedness condition.
- ▶ Open defaults can be handled differently (more model-theoretically).
- ▶ General proof methods for the finite, decidable case
- ▶ Applications of default logic:
  - ▶ Diagnosis
  - ▶ Reasoning about actions

# Literature



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