Qualitative Spatio-Temporal Reasoning with RCC-8 and Allen's Interval Calculus: Computational Complexity

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Abstract. There exist a number of qualitative constraint calculi that are used to represent and reason about temporal or spatial configurations. However, there are only very few approaches aiming to create a spatio-temporal constraint calculus. Similar to Bennett *et al.*, we start with the spatial calculus RCC-8 and Allen's interval calculus in order to construct a qualitative spatio-temporal calculus. As we will show, the basic calculus is NP-complete, even if we only permit base relations. When adding the restriction that the size of the spatial regions persists over time, or that changes are continuous, the calculus becomes more useful, but the satisfiability problem appears to be much harder. Nevertheless, we are able to show that satisfiability is still in NP.

1 Introduction

There exist a number of qualitative constraint calculi that are used to represent and reason about temporal or spatial configurations. For example, *Allen's* [1] *Interval Calculus* is certainly the most well-known qualitative temporal calculus in Artificial Intelligence. On the spatial side we have, for instance, the *Compass Calculus* [10], the generalization of Allen's interval calculus to two dimensions [2], and the topological *Region Connection Calculus* RCC-8 [15]. As pointed out by Wolter and Zakharyaschev [19], the next natural step would be to combine these two kinds of representation and reasoning.

Most of the existing proposals for spatio-temporal formalisms are more expressive than the above mentioned constraint calculi. Muller's [13] spatio-temporal theory is basically a first-order axiomatization of spatio-temporal entities based on RCC [15] and for this reason it is undecidable. Wolter and Zakharyaschev [19] combined the constraint formalism RCC-8 with the propositional temporal logic PTL [11]. This combination is very elegant because it can be expressed as a multi-modal logic based on Bennett's [3] encoding of RCC-8 as a multi-modal logic. However, the expressiveness of the resulting family of spatio-temporal formalisms is very high. Consequently, reasoning is PSPACE-hard for most of the proposed formalisms.

As mentioned by Wolter and Zakharyaschev [19], Allen's interval calculus is much closer in spirit to RCC-8 than PTL is. For this reason, it seems much more natural to use this calculus to "temporalize" RCC-8. A first attempt into this direction was done by Bennett *et al.* [4]. They provided the syntax and semantics of a combined calculus and embedded it into the combination of RCC-8 and PTL mentioned above. They also stated that the satisfiability problem of the combined calculus is NP-complete.

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Such a combined spatio-temporal formalism permits us to describe spatial configurations that change over time. We cannot state general laws of how the spatial configurations change, though. For example, we cannot state that regions cannot change their size, or that spatial changes should occur continuously. On the positive side, however, the restricted expressiveness results in only moderate computational requirements, as mentioned above. Nevertheless, we have a price to pay for the combination. Contrary to most other constraint formalisms, the spatio-temporal constraint formalism does not contain a computational tractable fragment that contains all basic relations and the universal relation.

Furthermore, the basic formalism does not address the issue of change in a reasonable way, but considers simply unrelated models over time. When adding constraints to the effect that changes have to be continuous, things become much more difficult. In particular, it is not obvious at all whether the satisfiability problem remains in NP. Using a large computer-generated case analysis and an inductive argument, we are able to show that satisfiability stays in NP. This result has the practical consequence that satisfiability can be solved by backtracking algorithms and other known techniques for NP-complete problems.

The rest of the paper is structured as follows. In the next section we give the necessary background on RCC-8 and Allen's Interval Calculus. In Section 3 we "temporalize" RCC-8 using Allen's interval calculus resulting in a *spatio-temporal constraint calculus* called STCC. In addition, we analyze the computational complexity of reasoning in this calculus and fragments thereof. In Section 4, we analyze the complexity of reasoning if it is required that the size of regions does not change. Finally, in Section 5, we analyze the case where all changes of spatial configurations happen continuously.

2 Background

RCC-8 is a well-known relation algebra for reasoning about binary relations between spatial regions in the context of the RCC-theory [15]. In this theory regions are non-empty regular, closed subsets of a topological space, and can consist of more than one piece. RCC-8 has eight *basic relations* which are jointly exhaustive and pairwise disjoint (see Figure 1): DC (DisConnected), EC (Externally Connected), PO (Partial Overlap), EQ (EQual), TPP (Tangential Proper Part), NTPP (Non-Tangential Proper Part), and their converse relations $\mathsf{TPP}^{\smile}$ and $\mathsf{NTPP}^{\smile}$. Each non-basic relation is the union of two or more basic relations, or the special *empty* relation. The set of RCC-8 relations corresponds to all possible subsets of the set of basic relations, where each subset is interpreted as the union of its relations. Hence, all in all, we have 2^8 different RCC-8 relations.

In Allen's Interval Calculus we reason about binary relations be-

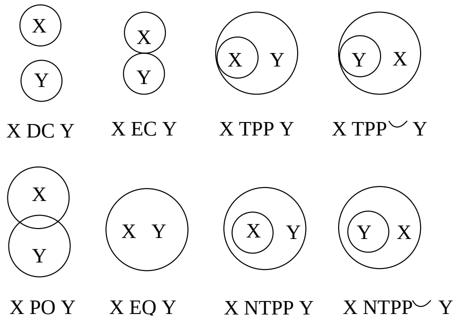

Figure 1. Two-dimensional examples for the eight basic relations of RCC-8

Relation	Converse	Pictorial Example
$I \prec J$	$J \succ I$	
I m J	J m$^{\smile}$ I	
I o J	J o$^{\smile}$ I	
I d J	J d$^{\smile}$ I	
I s J	J s$^{\smile}$ I	
I f J	J f$^{\smile}$ I	
I = J	J = I	

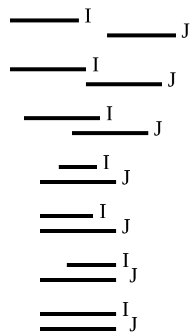

Figure 2. The thirteen basic relations of the Interval Algebra

tween intervals (over the time line, usually interpreted as the rationale numbers). These relations form again a relation algebra, which is called Interval Algebra (IA) [8]. IA has thirteen basic relations between intervals (see Figure 2): $\prec$ (before), m (meets), o (overlaps), d (during), s (starts), f (finishes), their converse relations ($\succ$, m$^{\smile}$, o$^{\smile}$, d$^{\smile}$, s$^{\smile}$, f$^{\smile}$), and = (equal). Like RCC-8, the relations forming IA correspond to all possible subsets of the set of basic relations.

RCC-8 and IA are closed under the following operations: union ($\cup$), intersection ($\cap$), difference ($\setminus$), converse ($^{\smile}$), and composition ($\circ$). The first four operations are defined in the standard way. Composition is just relational composition, i.e., for the relations r_1 and r_2, the composition $r_1 \circ r_2$ is defined as follows:

$$r_1 \circ r_2 = \{\langle x, y\rangle \mid \exists z\colon \langle x, z\rangle \in r_1, \langle z, y\rangle \in r_2\}.$$

A *spatial constraint satisfaction problem* (or briefly *spatial CSP*) in our context is a set Θ of atomic formulae (called *constraints*) of the kind XRY (using infix notation). X and Y are *region variables* and R is an RCC-8 relation. Similarly, a *temporal CSP* is a set of atomic formulae ISJ, where S is an IA relation and I, J are interval variables. If we do not want to distinguish between IA or RCC-8 formulae, we use the notation xry.

Given a spatial or temporal CSP Θ, a fundamental reasoning problem is deciding the *satisfiability* of Θ. Θ is satisfiable if and only if there is a *model* of Θ, i.e., an *assignment* of spatial regions or temporal intervals, respectively, to the variables of Θ such that all the constraints in Θ are satisfied. This problem is called RSAT for RCC-8 and ISAT for IA.

A related reasoning problem is finding a scenario that refines a given CSP Θ. A *scenario* is a satisfiable CSP, where the constraints between all pairs of variables are basic relations. Further, a CSP Θ' is a *refinement* of Θ if and only if Θ' and Θ involve the same variables, and for every pair of variables (x, y) such that $xR'y \in \Theta'$ and $xRy \in \Theta$, $R' \subseteq R$.[3]

Any spatial or temporal CSP Θ involving n variables can be processed using an $O(n^3)$ time algorithm that refines Θ to an equivalent *path consistent* CSP [12]. A CSP is path consistent if for every subset of constraints involving three variables i, j, and k, the relation R_{ik} between i and k is stronger or equal than (i.e., is a subset of) the composition of R_{ij} and R_{jk}.

From a computational point of view, RCC-8 and IA have some similar properties. Both RSAT and ISAT are NP-complete problems [17, 18], and several maximal tractable fragments have been identified both for RCC-8 and IA. Nebel and Bürckert [14] identified the unique maximal tractable sub-algebra of IA containing all the basic relations, which is called ORD-Horn class. ISAT for the ORD-Horn class can be decided in cubic time by using a path-consistency algorithm (a CSP over ORD-Horn is unsatisfiable if and only if the empty relation is generated when enforcing path-consistency). Other maximal tractable subclasses which do not contain all of the basic relations have been identified by Krokhin *et al.* [7].

Regarding RCC-8, Renz and Nebel identified three maximal tractable subclasses of RCC-8 containing all the basic relations [17, 16]. In addition, Gerevini and Renz [6] showed (using a technique called BIPATH-CONSISTENCY) that the relations in these maximal classes can be combined with qualitative size constraints without increasing the computational complexity.

Finally, RSAT and ISAT for the full RCC-8 and IA can be solved by finding a scenario through backtracking using path-consistency as a forward propagation technique [9].

3 Temporalizing RCC-8 Using Allen's Algebra

In order to *temporalize* RCC-8, we annotate spatial formulae with interval symbols. The intended meaning is that the spatial constraint is true during the interval denoted by the symbol. In other words, we can now write descriptions as follows:

$$\begin{array}{llll} I\colon & (X \quad \{\mathsf{DC}, \mathsf{EC}\} \quad Y), & I\colon & (Y \quad \{\mathsf{TPP}\} \quad Z), \\ J\colon & (X \quad \{\mathsf{PO}\} \quad Y), & J\colon & (Y \quad \{\mathsf{DC}\} \quad Z). \end{array}$$

This means that during interval I regions X and Y are disconnected or externally connected and that Y is a tangential proper part of Z. During J the spatial configuration is then a bit different. Of course, the next step is to use the IA relations to form IA constraints on interval symbols, e.g., $I\mathsf{m}J$. Sets of constraints on annotated spatial formulae and on intervals as in this example are called STCC CSPs.[4]

Having a closer look at this example, one notes that it seems to be unlikely that during one interval we have $(Y\{\mathsf{TPP}\}Z)$ and in the directly following interval we have $(Y\{\mathsf{DC}\}Z)$. Assuming that change happens *continuously*, one would expect that between I and J regions Y and Z are deformed and moved continuously, which implies that there are time spans between I and J when we have first $(Y\{\mathsf{PO}\}Z)$ and then $(Y\{\mathsf{EC}\}Z)$. However, before we deal with this issue, we will first lay the formal base for the combined constraint calculus.

[3] Without loss of generality we assume that if no information between x and y is provided, then R is the *universal relation*, i.e., the union of all the basic relations. Furthermore we assume that for every pair of variables (x, y) such that $xRy \in \Theta$, $yR^{\smile}x \in \Theta$.

[4] Bennett *et al.* [4] called a similar language **ARCC-8**.

As mentioned above, usually, temporal CSP variables are interpreted over pairs of the rational numbers $\mathbf{Q}$ and region variables are interpreted over non-empty, regular, closed subsets of some topological space $\mathcal{T}$, and we will follow this practice. A spatio-temporal interpretation is then a tuple $\mathcal{M} = (\mathbf{Q}, \mathcal{T}, \alpha)$, where α maps interval symbols to pairs of numbers from $\mathbf{Q}$, and region symbols and rational numbers to elements of $\mathcal{T}$. Such an interpretation is a *model of a* STCC *CSP* iff all temporal constraints are satisfied and each spatial formula annotated with an interval symbol I is satisfied at every point between the endpoints of the interval.[5] If such a model exists, we say that the STCC CSP is *satisfiable*. The associated reasoning problem is called RISAT.

Of course, for a given STCC CSP, we could try to construct a small, finite model, which could be extended to a full spatio-temporal model, in order to demonstrate satisfiability. We need, in fact, only polynomially many time points and associated RCC-8 models. Further, the RCC-8 models could be represented using Kripke models that have size polynomial in the size of the spatial CSP [17]. This implies the following proposition that has already been noted by Bennett *et al.* [4].

Proposition 1 RISAT *is* NP*-complete.*

This sounds like good news because it means that the complexity has not been increased by combining RCC-8 and IA. However, although the complexity is the same, we have a price to pay. While RCC-8 and IA each contains large fragments for which satisfiability can be decided in polynomial time (see above), this does not seem to be the case for STCC. In fact, the simple fragment in which we only have basic relations and the two universal relations is already NP-hard.

Theorem 2 RISAT *is* NP*-hard, even if the CSP contains only basic relations and the two universal relations.*

Proof. Consider the STCC CSP $\{I\colon(X\{\mathsf{DC}\}Y), J\colon(X\{\mathsf{EC}\}Y)\}$. This implies, however, $I\{\prec, \mathsf{m}, \mathsf{m}^{\smile}, \succ\}J$. The relation $\{\prec, \mathsf{m}, \mathsf{m}^{\smile}, \succ\}$ taken together with the basic temporal relations leads to the full algebra when closing the relations under intersection, converse, and composition [14], which implies that the satisfiability problem is NP-hard. ∎

Of course, if we refine every constraint between intervals and every constraint between regions inside an interval to a basic relation, then satisfiability becomes polynomial. However, such descriptions do not appear to be very useful. They imply that once two time intervals have more than their ending points in common, the two corresponding spatial scenarios must be identical over the full intervals. Conversely, if the spatial scenarios associated with different time intervals are pairwise different, only the relations $\prec, \succ, \mathsf{m}$, and $\mathsf{m}^{\smile}$ are possible between the intervals. For these reasons, such descriptions are probably hardly ever used in practice for describing spatio-temporal configurations. Nevertheless, such descriptions can be useful in the reasoning process. We will call descriptions which contain only spatial scenarios for each interval and the mentioned temporal relations between temporal intervals STCC-*scenarios*. Obviously, these scenarios will not necessarily arise by a refinement of the temporal and spatial constraints. However, if we have refined the temporal relations to a scenario, we get a totally ordered set of interval endpoints. This in turn can be used to define new temporal intervals, which can be ordered by $\prec, \succ, \mathsf{m}$, and $\mathsf{m}^{\smile}$.

As an example, assume that we have the temporal scenario $I \mathsf{o} J$. From this we get the ordering $\text{start}(I) < \text{start}(J) < \text{end}(I) < \text{end}(J)$. Based on that we can define the intervals $K = (\text{start}(I), \text{start}(J))$, $L = (\text{start}(J), \text{end}(I))$, and $M = (\text{end}(I), \text{end}(J))$, with the spatial constraints associated with I and J as constraints holding during K and M respectively, and with the intersection of the constraints associated with I and J as the constraints holding during L. Clearly this set of annotated spatial constraints combined with the (induced) temporal constraints $K \mathsf{m} J$ and $J \mathsf{m} M$ gives rise to a STCC CSP Θ that is satisfiable iff Θ admits a STCC scenario. STCC scenarios can obviously be used to generate STCC models. Hence, we can concentrate on generating such scenarios when we want to demonstrate satisfiability of a STCC CSP. In general, when Θ is a STCC CSP, we will call the CSP Θ' *induced* STCC scenario, if Θ' is generated from Θ in the way described above.

From the above, it follows that we will get a polynomial satisfiability problem, even if the CSP is not a STCC scenario.

Theorem 3 *For* STCC *CSPs where the temporal relations form a temporal scenario and the spatial relations are all elements of one tractable class of relations,* RISAT *is polynomial.*

4 The Size Persistence Constraint

As mentioned above, when we consider spatial scenarios changing over time, we may want to restrict the changes. For example, we may want to consider only changes where the regions *do not change their size*. Such restrictions cannot be stated inside the STCC formalism. However, we can, of course, restrict STCC models to those satisfying these restrictions.

Let $S(X)$ indicate the size of a spatial region X. Each RCC-8 constraint YRZ, where R is a basic relation, entails a qualitative size relation between Y and Z, which can be one of the following relations "$<$", "$>$", "$=$" or the indefinite relation "?". For instance, $(Y\{\mathsf{TPP}\}Z)$ entails $S(Y) < S(Z)$, while $(Y\{\mathsf{PO}\}Z)$ entails $S(Y)?S(Z)$. Table 1 gives the entailed size relation s for each basic RCC-8 relation R.

XRY		$S(X)sS(Y)$	XRY		$S(X)sS(Y)$
TPP	$\models$	$<$	EQ	$\models$	$=$
NTPP	$\models$	$<$	PO	$\models$	?
$\mathsf{TPP}^{\smile}$	$\models$	$>$	EC	$\models$	?
$\mathsf{NTPP}^{\smile}$	$\models$	$>$	DC	$\models$	?

Table 1. Size relations entailed by the basic RCC-8 relations

The *size persistence constraint* states that the size of every region persists over time, while their shape or relative position could change. For example, the following STCC CSP does not satisfy size persistence because the specified topological relations entail different size relations, regardless of the temporal relation between I and J:

$$I\colon\ (X\quad\{\mathsf{TPP}\}\quad Y),\qquad J\colon\ (X\quad\{\mathsf{EQ}\}\quad Y).$$

In general, we will require that there exist a scenario induced by the CSP that satisfies the condition that the relation between two regions does not conflict with respect to the derivable size relations. If this condition is satisfied, we say that the STCC CSP is satisfiable with respect to the size persistence constraint.

Theorem 4 RISAT *for a* STCC *CSP with the size persistence constraint is* NP*-complete.*

[5] In contrast to Bennett *et al.* [4] we do not require satisfaction at the endpoints because we want to allow that the spatial configurations change at endpoints of intervals.

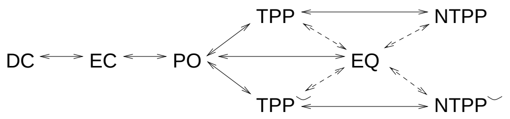

Figure 3. Neighborhood graph defining the continuous change of the basic RCC-8 relations. Dashed lines visualize the changes permitted if the size persistence constraint is not enforced.

Proof. NP-hardness obviously follows from the NP-completeness of RSAT for RCC-8 and of ISAT for IA. The following algorithm proves membership in NP. Guess a scenario Θ_t for the temporal CSP in the STCC CSP Θ under consideration, and then guess a spatial scenario for each sub-interval I of Θ_t. Check that the resulting set of spatial scenarios associated with Θ_t is an induced STCC scenario Θ' for Θ. In order to check that the persistence size constraint is satisfied, from the set of spatial scenarios of Θ' derive a new spatial CSP Θ_s, extended with size constraints, such that: for each spatial variable X of Θ and each interval I of Θ_t, X_I is a variable of Θ_s; for each each pair of variables X_I and Y_I, the constraint between X_I and Y_I is the constraint between X and Y in the scenario associated with I in Θ'; finally, for each pair of variables X_I and X_J such that $I \neq J$, $S(X_I) = S(X_J) \in \Theta_s$. Satisfiability of Θ_s can be checked in polynomial time by using BIPATH-CONSISTENCY [6]. ■

5 The Continuity Constraint

As mentioned, we may want to restrict spatial changes to those that are continuous. Similarly to the *size persistent constraint*, this cannot be expressed in the language itself, but it can be enforced as a condition on the models. Instead of requiring continuity of change on the models, we will be satisfied with a change of the relations between the models that is induced by a continuous change. For example, we may want to allow that a relation changes from DC to EC and then to PO, but a direct change from DC to PO is not allowed. Figure 3 gives a visualization of these changes, whereby the dashed lines visualize changes that are forbidden if the size persistence constraint has to be obeyed.

A path from r_1 to r_2 represents a multi-step transition, i.e., a sequence of continuous changes from relation r_1 to relation r_2. The set of all the paths in the graph represent all possible one-step or multi-step transitions. It is worth noting that, according to the semantics of STCC, during an interval of time I in the CSP, the spatial regions can be deformed and moved an arbitrary number of times changing the relative topological relation, and resulting into an arbitrary long sequence of spatial scenarios. What is important to preserve satisfiability of the CSP is that each of these configurations satisfies the formulae annotated with I in the CSP.

Moreover, under the continuous change restriction, each configuration should correspond to a scenario that can be modified only by transitions satisfying the neighborhood graph of Figure 3. For instance, if we have I: $(X\ \{\mathsf{DC}, \mathsf{EC}, \mathsf{TPP}\}\ Y)$, then during I the relation between X and Y can only change back and forth from DC to EC, or stay TPP (because a continuous change from, e.g., EC to TPP, requires that the relation is first PO, but this is forbidden by the given topological constraint).

This could make RISAT significantly harder, since the models of a STCC CSP might all involve an exponential number of RCC-8 models. In other words, it could be the case that in order to transform a scenario into another one by continuous changes, an exponential number of one-step transitions should happen, which would make membership in NP questionable at best. As we will show, fortunately this is not the case, at least under the size persistence restriction.

Given two scenarios σ_i and σ_f of a spatial CSP Σ, we define the *scenario transformation problem* as the problem of determining whether there exists a sequence $\sigma_1, ..., \sigma_n$ of scenarios for Σ such that $\sigma_1 = \sigma_i$, $\sigma_n = \sigma_f$, and the changes from σ_i to σ_{i+1} satisfy the neighborhood graph ($i = 1...n-1$). We call such a sequence a *transition chain* from σ_i to σ_f in Σ, and we indicate an instance of scenario transformation with a triple $\langle \Sigma, \sigma_i, \sigma_f \rangle$. Note that in a single step from σ_i to σ_{i+1} of a transition chain some changes regarding different pairs of regions must occur in parallel. For example, in order to change the scenario $\{X\{\mathsf{EQ}\}Y,\ Y\{\mathsf{EQ}\}Z,\ Z\{\mathsf{EQ}\}X\}$, it is necessary to simultaneously change two relations.

By using a large computer-generated case analysis we can prove that, for any solvable instance of scenario transformation involving three variables, under the size persistence restriction there exists a transition chain from σ_i to σ_f of fixed length. We conjecture that the same holds without the persistence size restriction, but in this paper we don't address this case. This program, which we will refer to as the *transition generator*, generates transition chains where, for every pair X, Y of variables and every basic RCC-8 relation R, XRY appears at most once in all scenarios of the chain. In other words, cyclic changes are not permitted. Furthermore, each scenario σ_{i+1} is derived from σ_i by performing a one-step transition to only one pair of regions, unless it is necessary to perform multiple parallel one-step transitions. When during the transformation of σ_i into σ_f we reach a scenario σ_j such that $X\{\mathsf{EQ}\}Y \in \sigma_j$ and $X\{\mathsf{EQ}\}Y \in \sigma_f$ ($j \neq f$), we collapse X and Y, and we proceed by considering changes only between either X and Z or Y and Z (further details are given in [5]). We call such transition chains from σ_i to σ_f *one-step transition chains*. The longest transition chains that are generated by the program have length 12. These chains are those solving the instances of scenario transformation where either

- in Σ the relation between every pair of variables is the universal relation or $\{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{TPP}, \mathsf{NTPP}\}$ and
$$\sigma_i = \{X\{\mathsf{DC}\}Y,\ Y\{\mathsf{DC}\}Z,\ Z\{\mathsf{DC}\}X\}$$
$$\sigma_f = \{X\{\mathsf{NTPP}\}Y,\ Y\{\mathsf{NTPP}\}Z,\ Z\{\mathsf{NTPP}\}X\},$$
- or in Σ the relation between every pair of variables is the universal relation or $\{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{TPP}, \mathsf{NTPP}^{\smile}\}$ and
$$\sigma_i = \{X\{\mathsf{DC}\}Y,\ Y\{\mathsf{DC}\}Z,\ Z\{\mathsf{DC}\}X\}$$
$$\sigma_f = \{X\{\mathsf{NTPP}^{\smile}\}Y,\ Y\{\mathsf{NTPP}^{\smile}\}Z,\ Z\{\mathsf{NTPP}^{\smile}\}X\}.$$

Hence, we can prove the following lemma that will be used to prove the main claim of this section.

Lemma 5 *Let Σ be a satisfiable spatial CSP involving three variables. If the scenario transformation $\langle \Sigma, \sigma_i, \sigma_f \rangle$ is solvable under the size persistence restriction, then in Σ there exists a one-step transition chain from σ_i to σ_f of at most 12 scenarios.*

Another property that can be verified by the transition generator is that, under certain conditions, there are only two types of parallel changes in any one-step transition chain. This property is exploited in the proof of the next lemma.

Proposition 6 *Let $\langle \Sigma, \sigma_i, \sigma_f \rangle$ be a scenario transformation that is solvable under the size persistence restriction. If there is no pair of variables X and Y such that $X\{\mathsf{EQ}\}Y \in \sigma_i$ and $X\{\mathsf{EQ}\}Y \in \sigma_f$, then in the transition chain computed by the transition generator every parallel change is of the form either $(X\{\mathsf{EQ}\}Y \rightarrow X\{\mathsf{PO}\}Y)$ or $(X\{\mathsf{PO}\}Y \rightarrow X\{\mathsf{EQ}\}Y)$.*

We now generalize Lemma 5 to CSPs involving an arbitrary number of variables, showing that any scenario transformation instance can be solved by transition chains of polynomial length, and thus that RISAT with continuous change and persistence size is in NP.[6]

Lemma 7 *Let $\langle \Sigma, \sigma_i, \sigma_f \rangle$ be any scenario transformation that is solvable under the size persistence restriction. Then in Σ there exists a transition chain from σ_i to σ_f of length less than $12 \cdot n^2$, where n is the number of variables in Σ.*

Proof Sketch. Consider every set V_j consisting of three different variables X_j,Y_j and Z_j appearing in Σ. Let Σ_j be the set of constraints in Σ involving the variables in V_j, σ_{ij} and σ_{if} the sub-scenarios of σ_i and σ_f, respectively, involving the variables in V_j, and $\mathcal{C}_j$ the one-step transition chain that is identified by the transition generator for $\langle \Sigma_j, \sigma_{ij}, \sigma_{fj} \rangle$. We show that by *synchronizing* all $\mathcal{C}_j$ we can find a transition chain from σ_i to σ_f in Σ involving less than $12 \cdot n^2$ transitions (scenarios of Σ).

There are two cases to consider, depending on whether there exists a k such that $\mathcal{C}_k$ involves a parallel transition. If there is no such a chain, then we can synchronize all the chains by running a topological sort algorithm on the *synchronization graph* G constructed as follows. The vertices of G are the constraints that are changed in one or more chains, and the edges correspond to the order of the changes. For instance, if a transition chain changes $c_1 = X\{\mathsf{DC}\}Y$, then $c_2 = X\{\mathsf{DC}\}Z$ and finally $c_3 = Y\{\mathsf{DC}\}Z$, G will contain the vertices c_1, c_2 and c_3, and edges from c_1 to c_2 and from c_2 to c_3. Vertices corresponding to the same change performed by different transition chains are collapsed into the same vertex, and the edges are appropriately updated.[7] From the the resulting topological sort we can derive a transition chain in Σ from σ_i to σ_f and, furthermore, by Lemma 5 this chain involves less than $12 \cdot n^2$ transitions.

For the case where there are parallel changes, we can use a similar argument, although the synchronization becomes more complicated, because parallel changes in a transition chain can imply further parallel changes that occur serialized in another chain. However, by exploiting Proposition 6 and Lemma 5 we can show that synchronization can still be accomplished by topological sort on a graph similar to G, where vertices represent sets of parallel changes and edges ordering constraints between them. ∎

Theorem 8 RISAT *for a* STCC *CSP with the continuous change and size persistence constraints is* NP*-complete.*

Proof Sketch. NP-hardness obviously follows from the NP-completeness of RSAT for RCC-8. Membership in NP can be proved by an argument similar to the one in the proof of Theorem 4, with the difference that instead of guessing one spatial scenario for each sub-interval I of Θ_t, we guess a sequence of $12 \cdot n^2$ scenarios. We check that they actually are all scenarios for the set of constraints associated with I, except for the first and the last one which can also be scenarios for the predecessor and the successor, respectively, sub-intervals. We then check that each sequence satisfies the continuity constraint. Lemma 7 guarantees that the number of scenarios in each sequence is sufficient. Finally, we check size persistence using a technique similar to the one used in the proof of Theorem 4. ∎

[6] Complete proofs and further details are available in [5].

[7] For instance, if there is another transition chain with the sequence of transitions changing $c'_1 = X\{\mathsf{EC}\}W$, then $c'_2 = X\{\mathsf{DC}\}Z$ and then $c'_3 = W\{\mathsf{DC}\}Z$, the vertices c_2 and c'_2 are collapsed and the destination (source) of all incoming (outgoing) edges involving c_2 or c'_2 is revised to the new collapsed vertex.

6 Summary and Conclusion

Similar to the approach by Bennett *et al.* [4], we temporalized RCC-8 using Allen's interval algebra IA. As we showed, satisfiability in the resulting calculus, called STCC, is NP-complete even if only basic relations and the two universal relations are permitted. Furthermore, we showed that the complexity does not increase if we additionally require changes to respect the size persistence and the continuity constraints.

While these results are quite useful and pave the ground for developing qualitative spatio-temporal reasoning algorithms, there remain a number of open questions that we intend to address in the near future. Firstly, it is not evident whether enforcing the continuity constraint guarantees that the regions can indeed change continuously. Secondly, it is not clear what *efficient* reasoning algorithms would look like. Such algorithms would most probably rely on forward-checking. Thirdly, there is the question for other reasonable restriction on spatial change, and how this could be incorporated in the constraint-reasoning framework.

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