Strengthening Landmark Heuristics via Hitting Sets

Blai Bonet[1] Malte Helmert[2]

[1]Universidad Simón Bolívar, Caracas, Venezuela

[2]Albert-Ludwigs-Universität Freiburg, Germany

July 23rd, 2010

Contribution

Our contribution

Area: heuristics for classical planning

Our contribution

- stronger way of exploiting landmarks for heuristic functions
- systematic way of generating landmarks for delete relaxation
- theoretical results relating new ideas to
  - admissible landmark heuristics (Karpas & Domshlak, 2009)
  - landmark-cut heuristic (Helmert & Domshlak, 2009)
  - optimal delete relaxation h^+ (Hoffmann & Nebel, 2001)
- new poly-time heuristic family that dominates landmark-cut
- ~~preliminary implementation and experiments~~

Relaxed planning

Optimal planning

Optimal planning:

- shortest paths in huge implicit graphs
- no formal definition here

What we need to know:

- state-of-the-art planners: heuristic search
- many use delete relaxation ("relaxed planning tasks")
- want accurate estimates of optimal delete relaxation cost h^+

Relaxed planning tasks

Definition (relaxed planning task)

F: finite set of facts

- initial facts $I \subseteq F$ are given
- goal facts $G \subseteq F$ must be reached
- operators of the form $o[4] : a, b \rightarrow c, d$

read: If we already have facts a and b (preconditions $pre(o)$),
we can apply o, paying 4 units (cost $cost(o)$),
to obtain facts c and d (effects $eff(o)$)

For simplicity: assume $I = \{i\}$, $G = \{g\}$, all $pre(o) \neq \emptyset$

Example: relaxed planning task

Example

$o_1[3] : i \rightarrow a, b$

$o_2[4] : i \rightarrow a, c$

$o_3[5] : i \rightarrow b, c$

$o_4[0] : a, b, c \rightarrow g$

Example: relaxed planning task

Example

$o_1[3] : i \rightarrow a, b$
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$o_4[0] : a, b, c \rightarrow g$

One way to reach $\{g\}$ from $\{i\}$:

- apply sequence o_1, o_2, o_4 (plan)
- cost: $3 + 4 + 0 = 7$ (optimal)

Optimal relaxed cost

- $h^+(I)$: minimal total cost to reach G from I
- NP-hard to compute (Bylander, 1994)
  or approximate by constant factor (Betz & Helmert, 2009)

$\leadsto$ use polynomial-time admissible heuristics

Landmarks

Landmarks

The most accurate current heuristics are based on landmarks.

Definition (landmark)

A (disjunctive action) landmark is a set of operators L such that each plan must contain some element of L.

The cost of a landmark, $cost(L)$, is $\min_{o \in L} \mathit{cost}(o)$.

$\rightsquigarrow$ the cost of any landmark is a (crude) admissible heuristic

Example: landmarks

Example

$o_1[3] : i \rightarrow a, b$

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Example: landmarks

Example

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Some landmarks:

- $W = \{o_4\}$ (cost 0)
- $X = \{o_1, o_2\}$ (cost 3)
- $Y = \{o_1, o_3\}$ (cost 3)
- $Z = \{o_2, o_3\}$ (cost 4)
- but also: $\{o_1, o_2, o_3\}$ (cost 3), $\{o_1, o_2, o_4\}$ (cost 0), ...

Exploiting landmarks

Exploiting landmarks

Assume we are given landmark set $\mathcal{L} = \{W, X, Y, Z\}$.
(later: how to find such landmarks)

How do we exploit $\mathcal{L}$ for heuristics?

- sum of costs $0 + 3 + 3 + 4 = 10 \rightsquigarrow$ inadmissible!
- maximum of costs: $\max\{0, 3, 3, 4\} = 4 \rightsquigarrow$ weak
- best previous approach: optimal cost partitioning

Landmark heuristics with optimal cost partitioning

optimal cost partitioning: Karpas & Domshlak (2009)

Idea: Derive a linear program (LP) from $\mathcal{L}$.

- one variable per landmark
- one constraint per operator

h^{L} value: objective value of the LP

Example: optimal cost partitioning

Example

$\mathit{cost}(o_1) = 3, \quad \mathit{cost}(o_2) = 4, \quad \mathit{cost}(o_3) = 5, \quad \mathit{cost}(o_4) = 0$

$\mathcal{L} = \{W, X, Y, Z\}$
with $W = \{o_4\}, \quad X = \{o_1, o_2\}, \quad Y = \{o_1, o_3\}, \quad Z = \{o_2, o_3\}$

LP: maximize $w + x + y + z$ subject to $w, x, y, z \geq 0$ and

$$\begin{array}{ccccccccc}
 & & x & + & y & & & \leq & 3 \\
 & & x & + & & & z & \leq & 4 \\
 & & & & y & + & z & \leq & 5 \\
w & & & & & & & \leq & 0
\end{array}$$

Example: optimal cost partitioning

Example

$cost(o_1) = 3$, $cost(o_2) = 4$, $cost(o_3) = 5$, $cost(o_4) = 0$

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 & & & y & + & z & \leq & 5 & o_3 \\
w & & & & & & \leq & 0 & o_4 \\
W & X & & Y & & Z & & &
\end{array}$$

Example: optimal cost partitioning

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w & & & & & & \leq & 0 & o_4 \\
W & X & & Y & & Z & & &
\end{array}$$

solution: $w = 0,\ x = 1,\ y = 2,\ z = 3 \quad \rightsquigarrow \quad h^{\mathsf{L}}(I) = 6$

Beyond optimal cost partitioning

- $h^{\mathsf{L}}(I) = 6$ is a good estimate, but $h^{+}(I) = 7$!
- Can we do better with the same information?

Hitting sets

Definition (hitting set)

Given: finite set A, subset family $\mathcal{F} \subseteq 2^A$, costs $c : A \to \mathbb{R}_0^+$

Hitting set:

- subset $H \subseteq A$ that "hits" all subsets in $\mathcal{F}$: $H \cap S \neq \emptyset$ for all $S \in \mathcal{F}$
- cost of H: $\sum_{a \in H} c(a)$

Minimum hitting set (MHS):

- minimizes cost
- classical NP-complete problem (Karp, 1972)

Example: hitting sets

Example

$A = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{W, X, Y, Z\}$
with $W = \{o_4\}, \;\; X = \{o_1, o_2\}, \;\; Y = \{o_1, o_3\}, \;\; Z = \{o_2, o_3\}$

$c(o_1) = 3, \;\; c(o_2) = 4, \;\; c(o_3) = 5, \;\; c(o_4) = 0$

Minimum hitting set:

Example: hitting sets

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$\mathcal{F} = \{W, X, Y, Z\}$
with $W = \{o_4\}, \;\; X = \{o_1, o_2\}, \;\; Y = \{o_1, o_3\}, \;\; Z = \{o_2, o_3\}$

$c(o_1) = 3, \;\; c(o_2) = 4, \;\; c(o_3) = 5, \;\; c(o_4) = 0$

Minimum hitting set: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting sets for landmarks

- can view landmark sets (with operator costs) as instances of minimum hitting set problem
- here, we got an admissible estimate that dominated h^{L}
- coincidence?

Hitting set heuristics

Let $\mathcal{L}$ be a set of landmarks.

Theorem (hitting set heuristics are admissible)

Let $h^{\mathsf{MHS}}(I)$ be the minimum hitting set cost for $\langle O, \mathcal{L}, \mathit{cost}\rangle$. Then:

1. $h^{\mathsf{MHS}}(I) \leq h^{+}(I)$ *(hitting set heuristics are admissible)*
2. $h^{\mathsf{MHS}}(I) \geq h^{\mathsf{L}}(I)$ *(hitting sets dominate cost partitioning)*

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Proof sketch:

1. plans are hitting sets (by definition of landmarks)
2. cost partitioning LP is dual of LP relaxation of hitting set integer program

Generating landmarks

Generating landmarks

How do we generate landmarks in the first place?

- most successful previous approach: LM-cut procedure (Helmert & Domshlak, 2009)
- here, we present a generalization

Justification graphs

Definition (precondition choice function)

A precondition choice function (pcf) $D : O \to F$
maps each operator to one of its preconditions.

Definition (justification graph)

The justification graph for pcf D is an arc-labeled digraph with

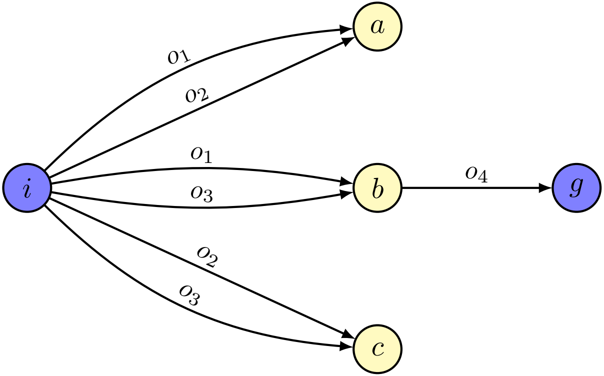
- vertices: the facts F
- arcs: arc $D(o) \xrightarrow{o} e$ for each operator o and effect $e \in \mathit{eff}(o)$

Example: justification graph

Example

pcf D: $D(o_1) = D(o_2) = D(o_3) = i$, $D(o_4) = b$

$o_1[3] : i \to a, b$
$o_2[4] : i \to a, c$
$o_3[5] : i \to b, c$
$o_4[0] : a, b, c \to g$



Cuts

Definition (cut)

A cut of a justification graph is a subset of its arcs C such that all paths from i to g use some arc in C.

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A cut of a justification graph is a subset of its arcs C such that all paths from i to g use some arc in C.

Theorem (cuts are landmarks)

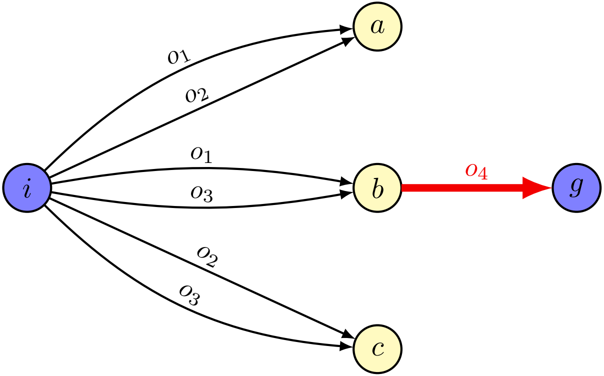
Let C be any cut of the justification graph for any pcf. Then the labels of C form a landmark.

Example: cuts of a justification graph

Example

Landmark $W = \{o_4\}$ (cost 0)

$o_1[3] : i \rightarrow a, b$
$o_2[4] : i \rightarrow a, c$
$o_3[5] : i \rightarrow b, c$
$o_4[0] : a, b, c \rightarrow g$



Example: cuts of a justification graph

Example

Landmark $X = \{o_1, o_2\}$ (cost 3)

$o_1[3] : i \rightarrow a, b$
$o_2[4] : i \rightarrow a, c$
$o_3[5] : i \rightarrow b, c$
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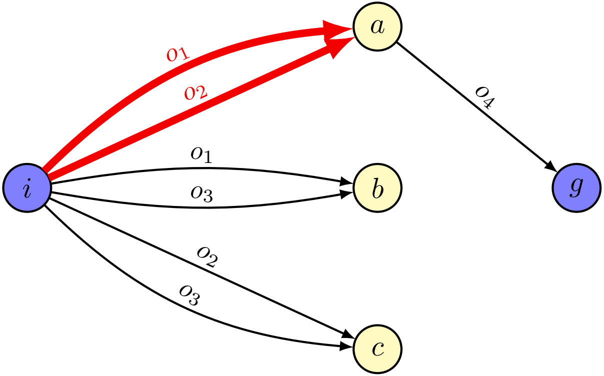


Example: cuts of a justification graph

Example

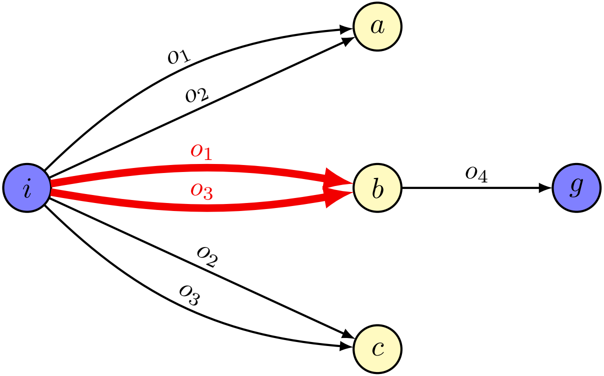
Landmark $Y = \{o_1, o_3\}$ (cost 3)

$o_1[3] : i \rightarrow a, b$

$o_2[4] : i \rightarrow a, c$

$o_3[5] : i \rightarrow b, c$

$o_4[0] : a, b, c \rightarrow g$

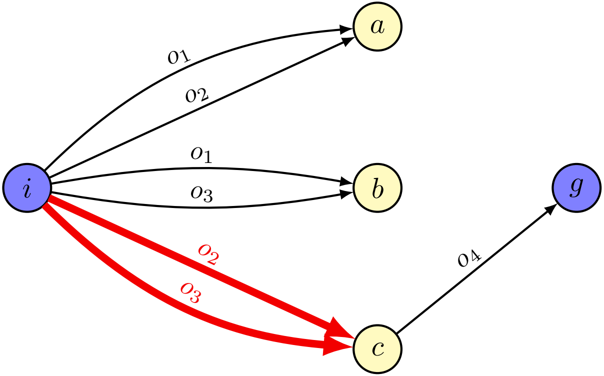


Example: cuts of a justification graph

Example

Landmark $Z = \{o_2, o_3\}$ (cost 4)

$o_1[3] : i \rightarrow a, b$
$o_2[4] : i \rightarrow a, c$
$o_3[5] : i \rightarrow b, c$
$o_4[0] : a, b, c \rightarrow g$



Power of justification graph cuts

- Which landmarks can be generated with the cut method?

Power of justification graph cuts

- Which landmarks can be generated with the cut method?
- All interesting ones!

Theorem (perfect hitting set heuristics)

Let $\mathcal{L}$ be the set of all "cut landmarks".
Then $h^{\mathit{MHS}}(I) = h^{+}(I)$.

$\rightsquigarrow$ hitting set heuristic over $\mathcal{L}$ is perfect

Power of justification graph cuts

- Which landmarks can be generated with the cut method?
- All interesting ones!

Theorem (perfect hitting set heuristics)

Let $\mathcal{L}$ be the set of all "cut landmarks".
Then $h^{\textit{MHS}}(I) = h^{+}(I)$.

$\rightsquigarrow$ hitting set heuristic over $\mathcal{L}$ is perfect

Proof sketch:

- We show that every hitting set H for $\mathcal{L}$ induces a plan.
- Assume that some hitting set H does not induce a plan.
- We construct a pcf and cut s.t. H does not hit the landmark.
- Contradiction!

Improving the LM-cut heuristic

Polynomial hitting set heuristics

How practical are our results?

- minimum hitting set is NP-hard
- number of cut landmarks is exponential

We now show how to apply our results to derive

- polynomial heuristics which
- dominate the LM-cut heuristic (Helmert & Domshlak, 2009).

LM-cut heuristic

$h^{\text{LM-cut}}$: Helmert & Domshlak (2009)

Initialize $h^{\text{LM-cut}}(I) := 0$. Then loop:

1. Compute h^{max} costs of facts. Stop if $h^{\text{max}}(g) = 0$.
2. Let D be a pcf that picks preconditions maximizing h^{max}.
3. Compute the justification graph for D.
4. Compute a cut using a particular procedure that guarantees that *cost*$(L) > 0$ for the induced landmark L.
5. Increase $h^{\text{LM-cut}}(I)$ by *cost*(L).
6. Decrease *cost*(o) by *cost*(L) for all $o \in L$.

Example: LM-cut computation

Example

round 1: $D(g) = a \rightsquigarrow L = \{o_2, o_3\}$ [4]

$o_1[3] : i \to a, b$

$o_2[4] : i \to a, c$

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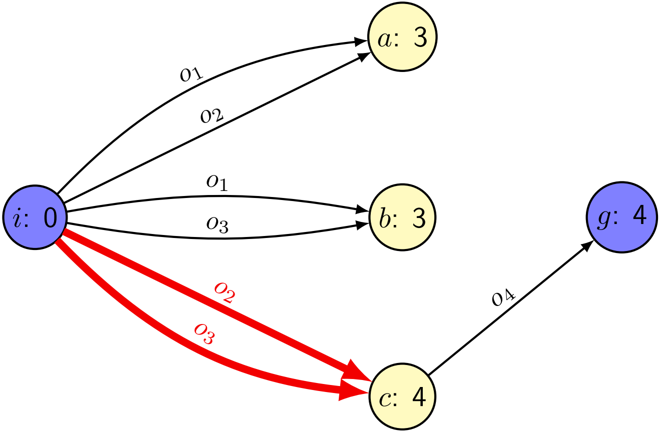

Example: LM-cut computation

Example

round 1: $D(g) = a \rightsquigarrow L = \{o_2, o_3\}$ [4] $\rightsquigarrow h^{\text{LM-cut}}(I) := 4$

$o_1[3] : i \rightarrow a, b$
$o_2[0] : i \rightarrow a, c$
$o_3[1] : i \rightarrow b, c$
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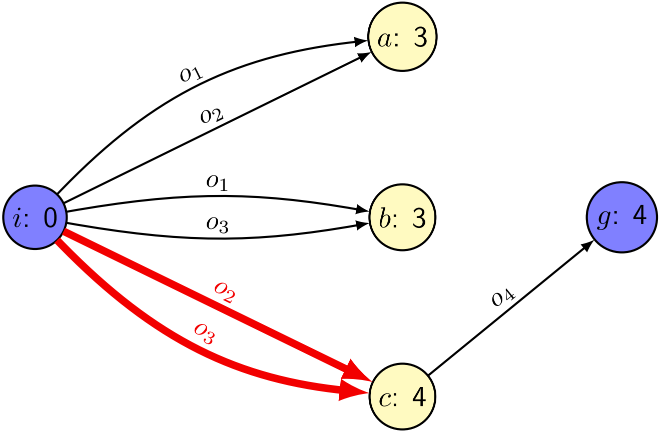


Example: LM-cut computation

Example

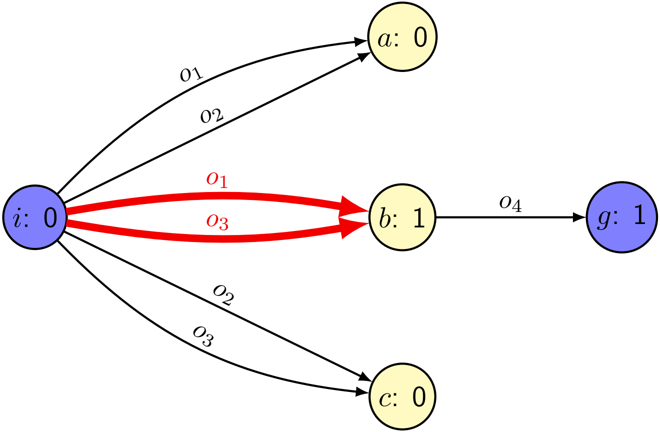
round 2: $D(g) = b \rightsquigarrow L = \{o_1, o_3\}$ [1]

$o_1[3] : i \to a, b$

$o_2[0] : i \to a, c$

$o_3[1] : i \to b, c$

$o_4[0] : a, b, c \to g$



Example: LM-cut computation

Example

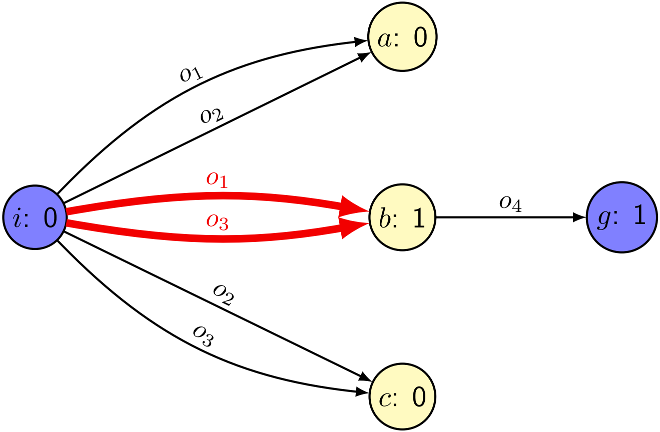
round 2: $D(g) = b \rightsquigarrow L = \{o_1, o_3\}$ [1] $\rightsquigarrow h^{\text{LM-cut}}(I) := 4 + 1 = 5$

$o_1[2] : i \rightarrow a, b$

$o_2[0] : i \rightarrow a, c$

$o_3[0] : i \rightarrow b, c$

$o_4[0] : a, b, c \rightarrow g$

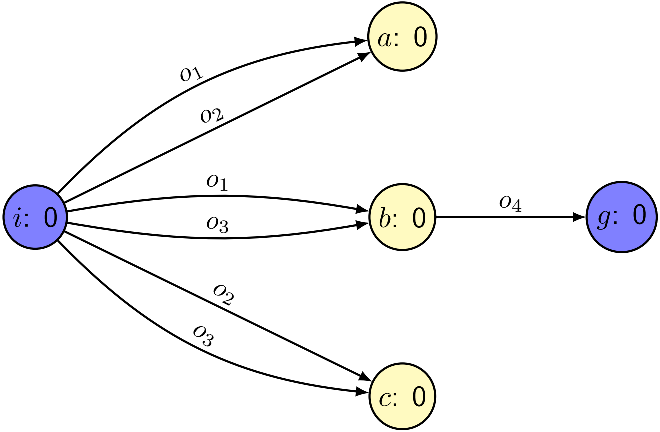


Example: LM-cut computation

Example

round 3: $h^{\text{max}}(g) = 0$ ⇝ done! ⇝ $h^{\text{LM-cut}}(I) = 5$

$o_1[2] : i \to a, b$
$o_2[0] : i \to a, c$
$o_3[0] : i \to b, c$
$o_4[0] : a, b, c \to g$



Improved LM-cut

We improve the LM-cut heuristic by

1. generating more landmarks, and
2. exploiting them in a smarter way.

Improved LM-cut: landmark generation

Generate more landmarks:

- Instead of performing the LM-cut computation once, perform it p times (p is a parameter).
- To make the runs different, use random tie-breaking when defining the h^{max}-based pcf.
- Collect all generated landmarks in a set $\mathcal{L}$.

Improved LM-cut: landmark exploitation

Exploit landmarks in a smarter way:

- We introduce a width parameter k for hitting set instances such that MHS is fixed-parameter tractable w.r.t. k.
- Remove some landmarks from $\mathcal{L}$ to bound the width.
- Solve resulting MHS problem in polynomial time.

Conclusion

Conclusion

Summary:

- Hitting sets for landmarks are more informative than optimal cost partitioning.
- Cuts in justification graphs offer a principled way of generating landmarks.
- Hitting sets over all cut landmarks are perfect heuristics for delete relaxations.
- These concepts can be exploited in practical heuristics.

What is next?

- Lots more!

⇝ Blai's talk(s) in late August/early September

The end

Thank you for your attention!