Strongest Postcondition

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- verification condition:
  strongest postcondition of ϕ under *C* entails ψ
  ($+$ special treatment of while)

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- next:
  static analysis constructs candidate for θ via forward analysis
  "reachability analysis"