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## Tutorial for Program Verification Exercise Sheet 9

Exercise 1: Soundness of the Hoare Proof System 0 Points

How can we prove the following Theorem of the lecture?

**Theorem** (Soundness of the Hoare proof system) If there is a derivation whose root is labelled by the Hoare triple  $\{\varphi\}st\{\psi\}$  then the statement st satisfies the precondition-postcondition pair  $(\{\varphi\}, \{\psi\})$ 

On Wednesday, we will discuss your ideas and carry out some parts of the proof in the lecture. Carrying out the remaining parts of the proof will become an exercise of Exercise Sheet 10.

**Exercise 2: While Rule of the Hoare Proof System** 2 Points

Use the Hoare proof system to show that the Hoare triple

$$\{ \substack{b \leq -23 \\ \land a \geq 42} \} \quad \texttt{while(!(b==0))} \{ \texttt{if(b>=0)} \{ \texttt{b}:=\texttt{b-1}; \} \texttt{else} \{ \texttt{b}:=\texttt{b+1}; \} \texttt{a}:=\texttt{a+1}; \} \quad \{ a \geq 47 \}$$

is valid. You may use that the following Hoare triple is valid. (I.e., you may use this Hoare triple as a leaf of the derivation tree.)

$$\{ {}^{(b \le 0 \to a - b \ge 53)}_{\vee(a \ge 37 \land b \le -17)} \} \quad \text{if(b>=0)} \{ \texttt{b:=b-1;} \} \texttt{else} \{ \texttt{b:=b+1;} \} \texttt{a:=a+1;} \quad \{ b \le 0 \to a - b \ge 53 \}$$

*Hint:* The while rule is not sufficient, you also need the rules (strepre) and (weakpos). The challenge is to find an inductive loop invariant  $\varphi$  that is implied by the precondition of the first Hoare triple above and that implies the precondition of the second Hoare triple above.