UNIVERSITÄT FREIBURG, INSTITUT FÜR INFORMATIK

SOFTWARETECHNIK SS01

ASSIGNMENT 6

Exercise 12: (10 Points)

Develop a Z-specification for the following part of the ECO-system. Interested persons can subscribe to special events. Each event is located in a room with a fixed number of seats. Therefore only a limited number of persons can subscribe. There is a waiting set for persons that have shown interest but could not yet be accepted. Specify the following operations: Persons can be added to and removed from the subscription and the waiting sets. There are further operations that show which persons have subscribed for a special event and which persons are on the waiting set. There is also an operation that returns all special events that a particular person has subscribed to.

Exercise 13: (10 Points)

(i) The least upper bound of two sets is the least set that contains each of the two sets. Give a generic definition in Z.

(ii) Show that the union of two sets is their least upper bound.

Show the following statements for relations R and S.

(iii) $\mathsf{dom}(R \cup S) = \mathsf{dom}\, R \cup \mathsf{dom}\, S$.

(iv) $\mathsf{dom}(R \cap S) \subseteq \mathsf{dom}\, R \cap \mathsf{dom}\, S$.

(v) $\mathsf{dom}\, R \cap \mathsf{dom}\, S \not\subseteq \mathsf{dom}(R \cap S)$.

(vi) $(R \fatsemi S)^{\sim} = S^{\sim} \fatsemi R^{\sim}$.

Hints:

- Use the Z-definitions at the following page.
- Use the following "elimination rule" for set comprehension.

$$y \in \{x_1 : X_1;\ x_2 : X_2;\ \ldots \mid P \bullet E\} \Leftrightarrow (\exists\, x_1 : X_1;\ x_2 : X_2;\ \ldots \bullet (P \wedge y = E)).$$

  Thereby P is a predicate, E an expression and y may neither occur in P nor be one of the x_i.
- Use the logical equivalences

$$\exists\, x.(\phi \vee \psi) \Leftrightarrow \exists\, x.\phi \vee \exists\, x.\psi, \qquad \exists\, x.(\phi \wedge \psi) \Rightarrow \exists\, x.\phi \wedge \exists\, x.\psi$$

Z-definitions:

- Set inclusion, union, intersection.

$$
\begin{array}{l}
[X] \\
\hline
\_ \subseteq \_ : \mathbb{P}\, X \leftrightarrow \mathbb{P}\, X \\
\_ \cup \_ : \mathbb{P}\, X \times \mathbb{P}\, X \rightarrow \mathbb{P}\, X \\
\_ \cap \_ : \mathbb{P}\, X \times \mathbb{P}\, X \rightarrow \mathbb{P}\, X \\
\hline
\forall\, S, T : \mathbb{P}\, X \bullet \\
\quad (S \subseteq T \Leftrightarrow (\forall\, x : X \bullet x \in S \Rightarrow x \in T)) \land \\
\quad S \cup T = \{x : X \mid x \in S \lor x \in T\} \land \\
\quad S \cap T = \{x : X \mid x \in S \land x \in T\}
\end{array}
$$

- Domain and range of a binary relation.

$$
\begin{array}{l}
[X, Y] \\
\hline
\mathsf{dom} : (X \leftrightarrow Y) \rightarrow \mathbb{P}\, X \\
\mathsf{ran} : (X \leftrightarrow Y) \rightarrow \mathbb{P}\, Y \\
\hline
\forall\, R : X \leftrightarrow Y \bullet \\
\quad \mathsf{dom}\, R = \{x : X;\ y : Y \mid x \underline{R} y \bullet x\} \land \\
\quad \mathsf{ran}\, R = \{x : X;\ y : Y \mid x \underline{R} y \bullet y\}
\end{array}
$$

thereby $x\underline{R}y \Leftrightarrow (x, y) \in R$.

- Relational composition.

$$
\begin{array}{l}
[X, Y, Z] \\
\hline
\_ \,⨾\, \_ : (X \leftrightarrow Y) \times (Y \leftrightarrow Z) \rightarrow (X \leftrightarrow Z) \\
\hline
\forall\, Q : X \leftrightarrow Y;\ R : Y \leftrightarrow Z \bullet \\
\quad Q \,⨾\, R = \{x : X;\ y : Y;\ z : Z \mid x \underline{Q} y \land y \underline{R} z \bullet x \mapsto z\}
\end{array}
$$

- Relational converse.

$$
\begin{array}{l}
[X, Y] \\
\hline
\_^{\sim} : (X \leftrightarrow Y) \rightarrow (Y \leftrightarrow X) \\
\hline
\forall\, R : X \leftrightarrow Y \bullet \\
\quad R^{\sim} = \{x : X;\ y : Y \mid x \underline{R} y \bullet y \mapsto x\}
\end{array}
$$

Please submit either a handwritten solution directly to your tutor or check in your solutions either as a postscript or a pdf document by July 12, 2001. A link to a reference to a LaTeX-package for Z can be found at the Softwaretechnik course web page.